UNIT – V
NETWORK SYNTHESIS

Topics: Identification of network synthesis-, Brune's positive and real function (PRF), properties of PRF, testing of driving point functions, even and odd function, one terminal pair network driving point synthesis with LC elements, RC elements, RL elements Foster and Cauer forms.

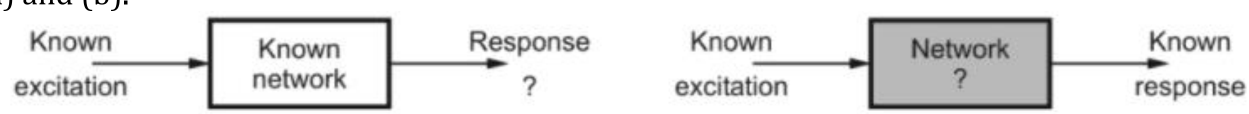
INTRODUCTION:

For any network, three things are associated with it. These are network elements, input i.e. excitation to the network and output i.e. response from the network.

In the **network analysis**, the network elements are known and excitation is also known. Using number of methods, the networks are studied and the response is obtained. Such a response is unique for a given network and known excitation. So obtaining a response for a known network and known excitation is called network analysis. Till now, number of methods and theorems are studied to analyze the network.

In the **network synthesis**, the procedure is exactly opposite to the analysis. The excitation is known and the response requirements are known. It is necessary to find the network satisfying the requirements. Thus obtaining a network for a known excitation and known response requirements is called **network synthesis**.

The basic difference between network analysis and synthesis is shown in the Fig. (a) and (b).



(a) Network analysis **(b) Network synthesis**

Another important difference between analysis and synthesis is that the analysis gives the solution i.e. response which is always unique. But synthesis may give different solutions i.e. networks satisfying the required specifications. Then the synthesis does not give us unique solution.

Then **Synthesis is the process of finding a network corresponding to a given driving point impedance or admittance (immittance = Impedance + admittance)**.

ELEMENTS OF REALIZABILITY

The starting point for any network synthesis problem is the network function N(s) which is the ratio of response R(s) to the excitation E(s).

Elements of realizability is the study to determine whether the network function could be realized as a physical passive network or not.

For any network function, there are two types of elements of realizability.

i) **Hurwitz Polynomial :** which is the denominator polynomial of the network function satisfying certain conditions.

ii) **Positive Real Function** : which is important because it represents physically realizable passive driving point impedance or admittances.

HURWITZ POLYNOMIAL

A polynomial P(s) is said to be Hurwitz if the following conditions are satisfied:
(i) P(s) is real when s is real.
(ii) The roots of P(s) have real parts which are zero or negative.

Properties of Hurwitz Polynomials

1. All the coefficients in the polynomial

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$$

are positive. A polynomial may not have any missing terms between the highest and the lowest order unless all even or all odd terms are missing.
2. The roots of odd and even parts of the polynomial P(s) lie on the jw-axis only.
3. If the polynomial P(s) is either even or odd, the roots of polynomial P(s) lie on the jw-axis only.
4. All the quotients are positive in the continued fraction expansion of the ratio of odd to even parts or even to odd parts of the polynomial P(s).
[Number of quotients = highest power of 's']
5. If the polynomial P(s) is expressed as W(s) P_1(s), then P(s) is Hurwitz if W(s) and P_1(s) are Hurwitz.
6. If the ratio of the polynomial P(s) and its derivative P′(s) gives a continued fraction expansion with all positive coefficients then the polynomial P(s) is Hurwitz.

PROBLEMS ON HURWITZ POLYNOMIAL

1) State for each case, whether the polynomial is Hurwitz or not. Give reasons in each case.
i) $s^4 + 4s^3 + 3s + 2$ *ii)* $s^6 + 5s^5 + 4s^4 - 3s^3 + 2s^2 + s + 3$

SOL:
i) In the given polynomial, the term s^2 is missing and it is neither an even nor an odd polynomial. Hence, it is not Hurwitz.
ii) Polynomial is not Hurwitz as it has a term ($-3s^3$) which has a negative coefficient.

2) Test whether the polynomial P(s) = $s^4 + s^3 + 5s^2 + 3s + 4$ is Hurwitz or not.
SOL:

$$\text{Even part of } P(s) = m(s) = s^4 + 5s^2 + 4$$

$$\text{Odd part of } P(s) = n(s) = s^3 + 3s$$

$$Q(s) = \frac{m(s)}{n(s)}$$

By continued fraction expansion,

$$s^3 + 3s)\,s^4 + 5s^2 + 4\ (s$$
$$\underline{s^4 + 3s^2}$$
$$2s^2 + 4\big)\,s^3 + 3s\left(\frac{1}{2}s\right.$$
$$\underline{s^3 + 2s}$$
$$s)\,2s^2 + 4\,(2s$$
$$\underline{2s^2}$$
$$4\big)\,s\left(\frac{1}{4}s\right.$$
$$\underline{s}$$
$$0$$

Since all the quotient terms are positive, P(s) is Hurwitz.

3) Test whether the polynomial P(s) = $s^3 + 4s^2 + 5s + 2$ is Hurwitz or not.
SOL:

$$\text{Even part of } P(s) = m(s) = 4s^2 + 2$$
$$\text{Odd part of } P(s)\ n(s) = s^3 + 5s$$

The continued fraction expansion can be obtained by dividing n(s) by m(s) as n(s) is of higher order than m(s).

$$Q(s) = \frac{n(s)}{m(s)}$$

$$4s^2 + 2\big)\,s^3 + 5s\left(\frac{1}{4}s\right.$$
$$\underline{s^3 + \frac{2}{4}s}$$
$$\frac{9}{2}s\big)\,4s^2 + 2\left(\frac{8}{9}s\right.$$
$$\underline{4s^2}$$
$$2\big)\,\frac{9}{2}s\left(\frac{9}{4}s\right.$$
$$\underline{\frac{9}{2}s}$$
$$0$$

Since all the quotient terms are positive, P(s) is Hurwitz.

4) Test whether the polynomial P(s) = $s^5 + 3s^3 + 2s$ is Hurwitz or not.
SOL:
Since the given polynomial contains odd functions only, it is not possible to perform a continued fraction expansion.

$$P'(s) = \frac{d}{ds}P(s) = 5s^4 + 9s^2 + 2$$

$$Q(s) = \frac{P(s)}{P'(s)}$$

By continued fraction expansion,

$$5s^4 + 9s^2 + 2 \overline{\big)\, s^5 + 3s^3 + 2s \,\big(} \frac{1}{5}s$$

$$\underline{s^5 + \frac{9}{5}s^3 + \frac{2}{5}s}$$

$$\frac{6}{5}s^3 + \frac{8}{5}s \,\big)\, 5s^4 + 9s^2 + 2 \,\big(\, \frac{25}{6}s$$

$$\underline{5s^4 + \frac{20}{3}s^2}$$

$$\frac{7}{3}s^2 + 2 \,\big)\, \frac{6}{5}s^3 + \frac{8}{5}s \,\big(\, \frac{18}{35}s$$

$$\underline{\frac{6}{5}s^3 + \frac{36}{35}s}$$

$$\frac{20}{35}s \,\big)\, \frac{7}{3}s^2 + 2 \,\big(\, \frac{49}{12}s$$

$$\underline{\frac{7}{3}s^2}$$

$$2 \,\big)\, \frac{20}{35}s \,\big(\, \frac{10}{35}s$$

$$\underline{\frac{20}{35}s}$$

$$0$$

Since all the quotient terms are positive, the polynomial P(s) is Hurwitz.

5) Test whether the polynomial P(s) = $2s^6 + s^5 + 13s^4 + 6s^3 + 56s^2 + 25s + 25$ is Hurwitz or not.
SOL:

$$\text{Even part of } P(s) = m(s) = 2s^6 + 13s^4 + 56s^2 + 25$$

$$\text{Odd part of } P(s) = n(s) = s^5 + 6s^3 + 25s$$

$$Q(s) = \frac{m(s)}{n(s)}$$

By continued fraction expansion,

$$
\begin{array}{l}
s^5+6s^3+25s)\,2s^6+13s^4+56s^2+25\,(2s \\
\qquad\qquad\qquad \underline{2s^6+12s^4+50s^2} \\
\qquad\qquad\qquad\qquad s^4+6s^2+25)\,s^5+6s^3+25s\,(s \\
\qquad\qquad\qquad\qquad\qquad\qquad\quad \underline{s^5+6s^3+25s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}
$$

The division has terminated abruptly.

$$P(s)=2s^6+s^5+13s^4+6s^3+56s^2+25s+25=(s^4+6s^2+25)\,(2s^2+s+1)$$

Let $\quad P_1(s)=s^4+6s^2+25$

Since $P_1(s)$ contains only even functions, we have to find the continued fraction expansion of $\dfrac{P_1(s)}{P_1'(s)}$.

$$P_1'(s)=4s^3+12s$$

By continued fraction expansion,

$$
\begin{array}{l}
4s^3+12s\,\big)\,s^4+6s^2+25\,\big(\tfrac{1}{4}s \\
\qquad\quad \underline{s^4+3s^2} \\
\qquad\qquad 3s^2+25\,\big)\,4s^3+12s\,\big(\tfrac{4}{3}s \\
\qquad\qquad\qquad\quad \underline{4s^3+\tfrac{100}{3}s} \\
\qquad\qquad\qquad\qquad -\tfrac{64}{3}s\,\big)\,3s^2+25\,\big(-\tfrac{9}{64}s \\
\qquad\qquad\qquad\qquad\qquad\quad \underline{3s^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad 25\,\big)\,-\tfrac{64}{3}s\,\big(-\tfrac{64}{75}s \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad \underline{-\tfrac{64}{3}s} \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}
$$

Since two of the quotient terms are negative, $P_1(s)$ is not Hurwitz.
We need not test the other factor $(2s^2+s+1)$ for being Hurwitz.
Hence, $P(s)$ is not Hurwitz.

6) Find the limits of *K* so that the polynomial $s^3 + 14s^2 + 56s + K$ may be Hurwitz.
SOL:

Odd part of the given polynomial, that is, $o(s) = s^3 + 56s$
Even part of the given polynomial, that is, $e(s) = 14s^2 + K$

The continued fraction expansion is given by the following:

$$14s^2 + K \overline{\big)\, s^3 + 56s \,\big(} \; \frac{s}{14}$$
$$s^3 + \frac{Ks}{14}$$
$$\left(56 - \frac{K}{14}\right)s \,\big)\, 14s^2 + K \,\big(\; \frac{14s}{56 - \frac{K}{14}}$$
$$14s^2$$
$$K \,\big)\, \left(56 - \frac{K}{14}\right)s \,\big(\; \frac{\left(56 - \frac{K}{14}\right)s}{K}$$
$$\left(56 - \frac{K}{14}\right)s$$
$$\text{x}$$

Now, for the polynomial to be Hurwitz, quotient terms should be positive.

That is, $$\frac{14}{56 - \frac{K}{14}} > 0$$

$$\frac{56 - \frac{K}{14}}{K} > 0$$

$$\frac{56 - \frac{K}{14}}{14} > \infty$$

$$56 - \frac{K}{14} > 0$$

This gives no value for *K*.

$$56 > \frac{K}{14}$$

$$K < 56 \times 14$$

$$K < 56 \times 14$$

$$K < 784$$

Therefore, the limit of *K* is $0 < K < 784$.

POSITIVE REAL FUNCTIONS (or) BRUNE'S POSITIVE REAL FUNCTIONS (PRF)

A function F(s) is positive real if the following conditions are satisfied:
(i) F(s) is real for real s.
(ii) The real part of F(s) is greater than or equal to zero when the real part of s is greater than or equal to zero, i.e., Re F(s) ≥ 0 for Re(s) ≥ 0.

Properties of Positive Real Functions

1. If F(s) is positive real then 1/F(s) is also positive real.
2. The sum of two positive real functions is positive real.
3. The poles and zeros of a positive real function cannot have positive real parts, i.e., they cannot be in the right half of the s plane.
4. Only simple poles with real positive residues can exist on the jw-axis.
5. The poles and zeros of a positive real function are real or occur in conjugate pairs.
6. The highest powers of the numerator and denominator polynomials may differ at most by unity. This condition prevents the possibility of multiple poles and zeros at s = ∞.
7. The lowest powers of the denominator and numerator polynomials may differ by at most unity. Hence, a positive real function has neither multiple poles nor zeros at the origin.

The necessary and sufficient conditions for a function with real coefficients F(s) to be positive real are the following:
1. F(s) must have no poles and zeros in the right half of the s-plane.
2. The poles of F(s) on the jω-axis must be simple and the residues evaluated at these poles must be real and positive.
3. Re F (jω) ≥ 0 for all ω.

REQUIREMENTS FOR POSITIVE REALNESS OF SIMPLE RATIONAL POLYNOMIAL QUOTIETNS

S.No.	Type of Function, F(s)	Requirement for F(s) to be Positive Real
1.	$F(s) = \dfrac{s+\beta}{s+\alpha}, \quad \alpha, \beta\ real$	$\boldsymbol{\alpha}, \boldsymbol{\beta} \geqslant 0$
2.	$F(s) = \dfrac{ks}{s^2+\alpha}, \quad \alpha, k\ real$	$\boldsymbol{\alpha}$, k $\geqslant 0$
3.	$F(s) = \dfrac{s+a}{s^2+bs+c}, \quad a, b, c\ real$	i) a,b,c $\geqslant 0$ ii) b $\geqslant$ a
4.	$F(s) = \dfrac{s^2+a_1 s+a_0}{s^2+b_1 s+b_0}, \quad a_0, a_1, b_0, b_1\ real$	i) $a_0, a_1, b_0, b_1 \geqslant 0$ $ii)\ a_1 b_1 \geq \left[\sqrt{a_0}-\sqrt{b_0}\right]^2$

EVEN AND ODD PARTS OF A FUNCTION F(s)

Let a function

$$F(s) = \frac{P(s)}{Q(s)}$$

Separate even and odd parts

$$F(s) = \frac{e_1(s) + o_1(s)}{e_2(s) + o_2(s)}$$

Divide and multiply with $e_2(s) - o_2(s)$

$$F(s) = \frac{e_1(s) + o_1(s)}{e_2(s) + o_2(s)} \times \frac{e_2(s) - o_2(s)}{e_2(s) - o_2(s)}$$

$$Even[F(s)] = \text{Re}[F(j\omega)] = \frac{e_1(s)\, e_2(s) - o_1(s)\, o_2(s)}{[e_2(s)]^2 - [o_2(s)]^2}$$

$$Odd[F(s)] = j\text{Im}[F(j\omega)] = \frac{e_2(s)\, o_1(s) - e_1(s)\, o_2(s)}{[e_2(s)]^2 - [o_2(s)]^2}$$

PROBLEMS ON POSITIVE REAL FUNCTIONS

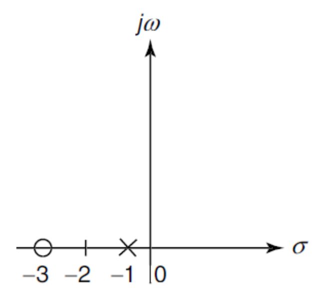
1) Test whether $F(s) = \dfrac{s+3}{s+1}$ is a positive real or not.

SOL:

(a) $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s+3}{s+1}$

The function $F(s)$ has pole at $s = -1$ and zero at $s = -3$ as shown in Fig.

Thus, pole and zero are in the left half of the s-plane.



(b) There is no pole on the $j\omega$ axis. Hence, the residue test is not carried out.

(c) Even part of $N(s) = m_1 = 3$

Odd part of $N(s) = n_1 = s$

Even part of $D(s) = m_2 = 1$

Odd part of $D(s) = n_2 = s$

$$A(\omega^2) = m_1 m_2 - n_1 n_2 \,|_{s=j\omega} = (3)(1) - (s)(s)\,|_{s=j\omega} = 3 - s^2\,|_{s=j\omega} = 3 + \omega^2$$

$A(\omega^2)$ is positive for all $\omega \geq 0$.

Since all the three conditions are satisfied, the function is positive real.

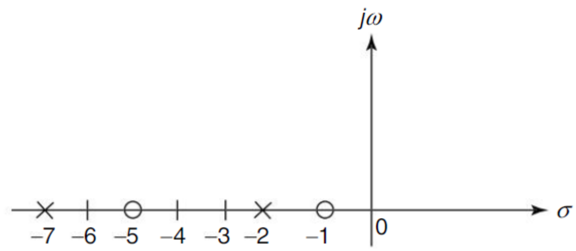
2) Test whether $F(s) = \dfrac{s^2+6s+5}{s^2+9s+14}$ is positive real function or not.

SOL:

(a) $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s^2+6s+5}{s^2+9s+14} = \dfrac{(s+5)(s+1)}{(s+7)(s+2)}$

The function $F(s)$ has poles at $s=-7$ and $s=-2$ and zeros at $s=-5$ an $s=-1$ as shown in Fig.

Thus, all the poles and zeros are in the left half of the s plane.



(b) Since there is no pole on the $j\omega$ axis, the residue test is not carried out.

(c) Even part of $N(s) = m_1 = s^2+5$

Odd part of $N(s) = n_1 = 6s$

Even part of $D(s) = m_2 = s^2+14$

Odd part of $D(s) = n_2 = 9s$

$$A(\omega^2) = m_1m_2 - n_1n_2 \big|_{s=j\omega} = (s^2+5)(s^2+14) - (6s)(9s) \big|_{s=j\omega} = s^4 - 35s^2 + 70 \big|_{s=j\omega} = \omega^4 + 35\omega^2 + 70$$

$A(\omega^2)$ is positive for all $\omega \geq 0$.

Since all the three conditions are satisfied, the function is positive real.

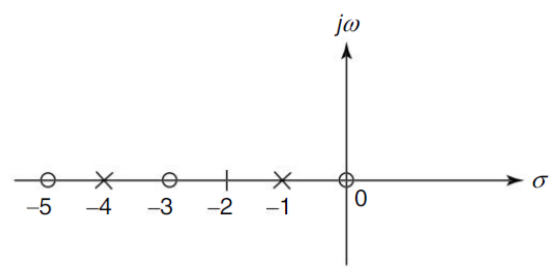
3) Test whether $F(s) = \dfrac{s(s+3)(s+5)}{(s+1)(s+4)}$ is positive real function or not.

SOL:

(a) $F(s) = \dfrac{N(s)}{D(s)} = \dfrac{s(s+3)(s+5)}{(s+1)(s+4)} = \dfrac{s^3+8s^2+15s}{s^2+5s+4}$

The function $F(s)$ has poles at $s=-1$ and $s=-4$ and zeros at $s=0$, $s=-3$ and $s=-5$ as shown in Fig.

Thus, all the poles and zeros are in the left half of the s plane.



(b) There is no pole on the $j\omega$ axis, hence the residue test is not carried out.

(c) Even part of $N(s) = m_1 = 8s^2$

Odd part of $N(s) = n_1 = s^3+15s$

Even part of $D(s) = m_2 = s^2+4$

Odd part of $D(s) = n_2 = 5s$

$$A(\omega^2) = m_1m_2 - n_1n_2 \big|_{s=j\omega} = (8s^2)(s^2+4) - (s^3+15s)(5s) \big|_{s=j\omega} = 3s^4 - 43s^2 \big|_{s=j\omega} = 3\omega^4 + 43\omega^2$$

$A(\omega^2)$ is positive for all $\omega \geq 0$.

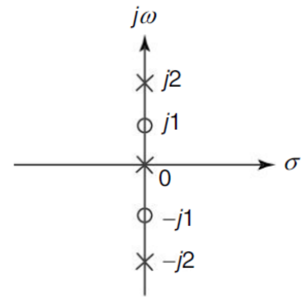
Since all the three conditions are satisfied, the function is positive real.

4) Test whether $F(s) = \dfrac{s^2+1}{s^3+4s}$ is positive real function or not.

SOL:

(a) $$F(s) = \frac{N(s)}{D(s)} = \frac{s^2+1}{s^3+4s} = \frac{(s+j1)(s-j1)}{s(s+j2)(s-j2)}$$

The function $F(s)$ has poles at $s = 0$, $s = -j2$ and $s = j2$ and zeros at $s = -j1$ and $s = j1$ as shown in Fig.



Thus, all the poles and zeros are on the $j\omega$ axis.

(b) The poles on the $j\omega$ axis are simple. Hence, residue test is carried out.

$$F(s) = \frac{s^2+1}{s^3+4s} = \frac{s^2+1}{s(s^2+4)}$$

By partial-fraction expansion,

$$F(s) = \frac{K_1}{s} + \frac{K_2}{s+j2} + \frac{K_2^*}{s-j2}$$

The constants K_1, K_2 and K_2* are called residues.

$$K_1 = s\,F(s)\,|_{s=0} = \left.\frac{s^2+1}{s^2+4}\right|_{s=0} = \frac{1}{4}$$

$$K_2 = (s+j2)F(s)\,|_{s=-j2} = \left.\frac{s^2+1}{s(s-j2)}\right|_{s=-j2} = \frac{-4+1}{(-j2)(-j2-j2)} = \frac{3}{8}$$

$$K_2^* = K_2 = \frac{3}{8}$$

Thus, residues are real and positive.

(c) Even part of $N(s) = m_1 = s^2+1$

Odd part of $N(s) = n_1 = 0$

Even part of $D(s) = m_2 = 0$

Odd part of $D(s) = n_2 = s^3+4s$

$$A\left(\omega^2\right) = m_1 m_2 - n_1 n_2\,|_{s=j\omega} = (s^2+1)(0) - (0)(s^3+4s)\,|_{s=j\omega} = 0$$

$A(\omega^2)$ is zero for all $\omega \geq 0$.

Since all the three conditions are satisfied, the function is positive real.

5) Test whether $F(s) = \dfrac{s^2+s+6}{s^2+s+1}$ is positive real function or not.

SOL:

$$\text{(a)}\quad F(s) = \frac{N(s)}{D(s)} = \frac{s^2+s+6}{s^2+s+1} = \frac{\left(s+\frac{1}{2}+j\frac{\sqrt{23}}{2}\right)\left(s+\frac{1}{2}-j\frac{\sqrt{23}}{2}\right)}{\left(s+\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)\left(s+\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)}$$

The function $F(s)$ has zeros at $s = -\frac{1}{2} \pm j\frac{\sqrt{23}}{2}$ and poles at $s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$.

(b) There is no pole on the $j\omega$ axis. Hence, the residue test is not carried out.

(c) Even part of $N(s) = m_1 = s^2 + 6$

Odd part of $N(s) = n_1 = s$

Even part of $D(s) = m_2 = s^2 + 1$

Odd part of $D(s) = n_2 = s$

$$A(\omega^2) = m_1 m_2 - n_1 n_2 \big|_{s=j\omega} = (s^2+6)(s^2+1) - (s)(s) \big|_{s=j\omega} = s^4 + 6s^2 + 6 \big|_{s=j\omega} = \omega^4 - 6\omega^2 + 6$$

For $\omega = 2$, $A(\omega^2) = 16 - 24 + 6 = -2$

This condition is not satisfied.

Hence, the function $F(s)$ is not positve real.

ELEMENTARY SYNTHESIS CONCEPTS

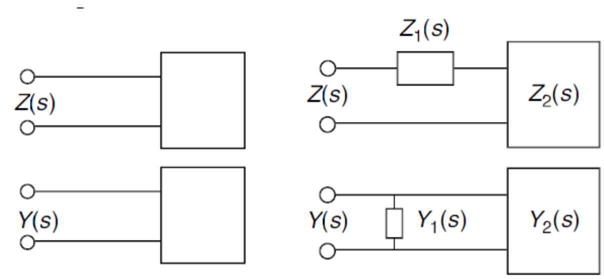
We know that impedances and admittances of passive networks are positive real functions. Hence, addition of impedances of the two passive networks gives a function which is also a positive real function. Thus, $Z(s) = Z_1(s) + Z_2(s)$ is a positive real function, if $Z_1(s)$ and $Z_2(s)$ are positive real functions. Similarly, $Y(s) = Y_1(s) + Y_2(s)$ is a positive real function, if $Y_1(s)$ and $Y_2(s)$ are positive real functions. There is a special terminology for synthesis procedure. We have,

$$Z(s) = Z_1(s) + Z_2(s)$$

$$Z_2(s) = Z(s) - Z_1(s)$$

Here, $Z_1(s)$ is said to have been removed from $Z(s)$ in forming the new function $Z_2(s)$ as shown in Fig. If the removed network is associated with the pole or zero of the original network impedance then that pole or zero is also said to have been removed.

There are four important removal operations.



1) Removal of a Pole at Infinity

Consider an impedance function $Z(s)$ having a pole at infinity which means that the numerator polynomial is one degree greater than the degree of the denominator polynomial.

$$Z(s) = \frac{a_{n+1}s^{n+1} + a_n s^n + \ldots + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0} = Hs + \frac{c_n s^n + c_{n-1}s^{n-1} + \ldots + c_1 s + c_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}$$

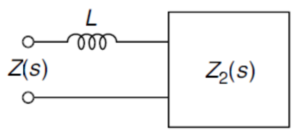
where $H = \frac{a_{n+1}}{b_n}$

Let $Z_1(s) = Hs$

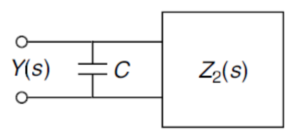
and $Z_2(s) = \frac{c_n s^n + c_{n-1}s^{n-1} + \ldots + c_1 s + c_0}{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0} = Z(s) - Hs$

$Z_1(s) = Hs$ represents impedance of an inductor of value H. Hence, the removal of a pole at infinity corresponds to the removal of an inductor from the network of Fig.(a).

If the given function is an admittance function $Y(s)$, then $Y_1(s) = Hs$ represents the admittance of a capacitor $Y_C(s) = Cs$. The network for $Y_1(s)$ is a capacitor of value $C = H$ as shown in Fig.(b).



(a)



(b)

2) Removal of a Pole at Origin

If $Z(s)$ has a pole at the origin then it may be written as

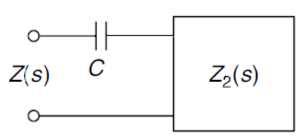
$$Z(s) = \frac{a_0 + a_1 s + \ldots + a_{n-1}s^{n-1} + a_n s^n}{b_1 s + b_2 s^2 + \ldots + b_m s^m} = \frac{K_0}{s} + \frac{d_1 + d_2 s + \ldots + d_n s^{n-1}}{b_1 + b_2 s + \ldots + b_m s^{m-1}} = Z_1(s) + Z_2(s)$$

where $K_0 = \frac{a_0}{b_1}$

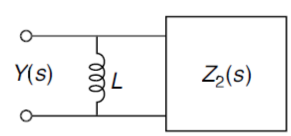
$Z_1(s) = \frac{K_0}{s}$ represents the impedance of a capacitor of value $\frac{1}{K_0}$.

If the given function is an admittance function $Y(s)$ then removal of $Y_1(s) = \frac{K_0}{s}$ corresponds to an inductor of value $\frac{1}{K_0}$.

Thus, removal of a pole from the impedance function $Z(s)$ at the origin corresponds to the removal of a capacitor, and from admittance function $Y(s)$ corresponds to removal of an inductor as shown in Fig.



(a)



(b)

3) Removal of Conjugate Imaginary Poles

If $Z(s)$ contains poles on the imaginary axis, i.e., at $s = \pm j\omega_1$ then $Z(s)$ will have factors $(s + j\omega_1)(s - j\omega_1) = s^2 + \omega_1^2$ in the denominator polynomial

$$Z(s) = \frac{p(s)}{(s^2 + \omega_1^2)\, q_1(s)}$$

By partial-fraction expansion,

$$Z(s) = \frac{K_1}{s + j\omega_1} + \frac{K_1^*}{s - j\omega_1} + Z_2(s)$$

For a positive real function, $j\omega$ axis poles must themselves be conjugate and must have equal, positive and real residues.

$$K_1 = K_1^*$$

Hence,
$$Z(s) = \frac{2K_1 s}{s^2 + \omega_1^2} + Z_2(s)$$

Thus,
$$Z_1(s) = \frac{2K_1 s}{s^2 + \omega_1^2} = \frac{1}{\dfrac{s}{2K_1} + \dfrac{\omega_1^2}{2K_1 s}} = \frac{1}{Y_a + Y_b}$$

where $Y_a = \dfrac{s}{2K_1}$ is the admittance of a capacitor of value $C = \dfrac{1}{2K_1}$

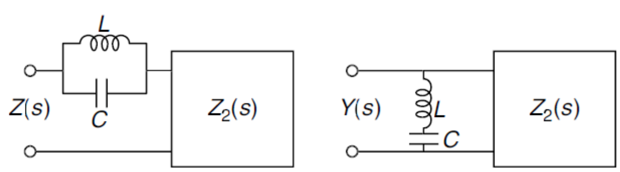
and $Y_b = \dfrac{\omega_1^2}{2K_1 s}$ is the admittance of an inductor of value $L = \dfrac{2K_1}{\omega_1^2}$

If the given function is an admittance function $Y(s)$ then

$$Y_1(s) = \frac{2K_1 s}{s^2 + \omega_1^2} = \frac{1}{Z_a + Z_b} = \frac{1}{\dfrac{s}{2K_1} + \dfrac{\omega_1^2}{2K_1 s}}$$

where $Z_a = \dfrac{s}{2K_1}$ is the impedance of an inductor of value $L = \dfrac{1}{2K_1}$ and $Z_b = \dfrac{\omega_1^2}{2K_1 s}$ is the impedance of a capacitor of value $C = \dfrac{2K_1}{\omega_1^2}$.

Thus, removal of conjugate imaginary poles from impedance function $Z(s)$ corresponds to the removal of the parallel combination of $L - C$ and from admittance function $Y(s)$ corresponds to removal of series combination of $L - C$ as shown in Fig.



4) Removal of a Constant

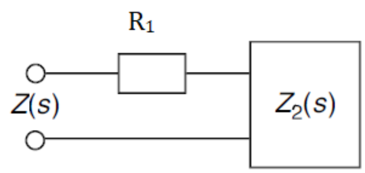
If a real number R_1 is subtracted from $Z(s)$ such that

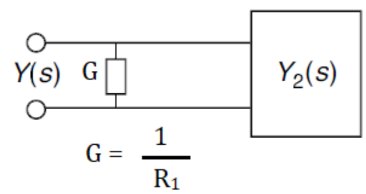
$$Z_2(s) = Z(s) - R_1$$
$$Z(s) = R_1 + Z_2(s)$$

then R_1 represents a resistor.

If the given function is an admittance function $Y(s)$, then removal of $Y_1(s) = R_1$ represents a conductance of value R_1.

Thus, removal of a constant from impedance function $Z(s)$ corresponds to the removal of a resistance, and from admittance function $Y(s)$ corresponds to removal of a conductance.





PROBLEMS ON REALIZATION OF NETWOTK FUNCTIONS

1) Synthesize the impedance function $Z(s) = \dfrac{s^3 + 4s}{s^2 + 2}$.

SOL:

By long division of $Z(s)$,

$$s^2+2\,\big)\,s^3+4s\,\big(\,s$$
$$\underline{s^3+2s}$$
$$2s$$

$$Z(s) = s + \frac{2s}{s^2+2} = Z_1(s) + Z_2(s)$$

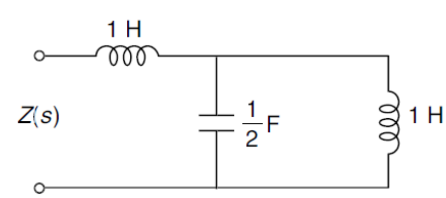
$Z_1(s) = s$ represents impedance of an inductor of value 1 H.

$$Y_2(s) = \frac{1}{Z_2(s)} = \frac{s^2+2}{2s} = \frac{s^2}{2s} + \frac{2}{2s} = \frac{1}{2}s + \frac{1}{s} = Y_3(s) + Y_4(s)$$

$Y_3(s) = \frac{1}{2}s$ represents the admittance of a capacitor of value $\frac{1}{2}$ F.

$Y_4(s) = \frac{1}{s}$ represents the admittance of an inductor of value 1 H.

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches. The network is shown in Fig.



2) Realize the network having impedance function $Z(s) = \dfrac{6s^3 + 5s^2 + 6s + 4}{2s^3 + 2s}$.

SOL:

By long division of $Z(s)$,

$$2s^3 + 2s \,\big)\, 6s^3 + 5s^2 + 6s + 4 \,\big(\, 3$$
$$\underline{6s^3 \qquad + 6s}$$
$$5s^2 \qquad + 4$$

$$Z(s) = 3 + \frac{5s^2 + 4}{2s^3 + 2s} = Z_1(s) + Z_2(s)$$

$Z_1(s) = 3$ represents the impedance of a resistor of value 3 Ω.

$$Y_2(s) = \frac{1}{Z_2(s)} = \frac{2s^3 + 2s}{5s^2 + 4}$$

By long division of $Y_2(s)$,

$$5s^2 + 4 \,\big)\, 2s^3 + 2s \,\big(\, \frac{2}{5}s$$
$$\underline{2s^3 + \frac{8}{5}s}$$
$$\frac{2}{5}s$$

$$Y_2(s) = \frac{2}{5}s + \frac{\frac{2}{5}s}{5s^2 + 4} = Y_3(s) + Y_4(s)$$

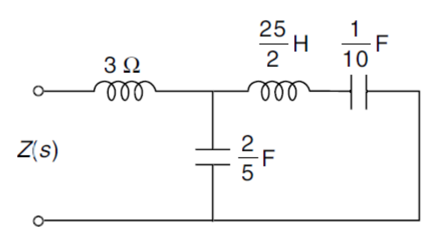
$Y_3(s) = \dfrac{2}{5}s$ represents the admittance of a capacitor of value $\dfrac{2}{5}$F.

$$Z_4(s) = \frac{1}{Y_4(s)} = \frac{5s^2 + 4}{\frac{2}{5}s} = \frac{25s^2 + 20}{2s} = \frac{25}{2}s + \frac{10}{s} = Z_5(s) + Z_6(s)$$

$Z_5(s) = \dfrac{25}{2}s$ represents the impedance of an inductor of value $\dfrac{25}{2}$H.

$Z_6(s) = \dfrac{10}{s}$ represents the impedance of a capacitor of value $\dfrac{1}{10}$F.

The impedances are connected in the series branches, whereas the admittances are connected in the parallel branches. The network is shown in Fig.



3) Realize the network having admittance function $Y(s) = \dfrac{4s^2 + 6s}{s+1}$

SOL:

By long division of $Y(s)$,

$$s+1 \overline{)\, 4s^2 + 6s \,(} \; 4s$$
$$\underline{4s^2 + 4s}$$
$$2s$$

$$Y(s) = 4s + \frac{2s}{s+1} = Y_1(s) + Y_2(s)$$

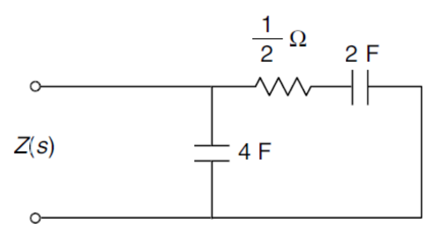
$Y_1(s) = 4s$ represents the admittance of a capacitor of value 4 F.

$$Z_2(s) = \frac{1}{Y_2(s)} = \frac{s+1}{2s} = \frac{1}{2} + \frac{1}{2s} = Z_3(s) + Z_4(s)$$

$Z_3(s) = \frac{1}{2}$ represents the impedance of a resistor of value $\frac{1}{2}\,\Omega$.

$Z_4(s) = \frac{1}{2s}$ represents the impedance of a capacitor of value 2 F.

The impedances are connected in the series branches, whereas the admittances are connected in the parallel branches. The network is shown in Fig.



4) Realize the network having admittance function $Y(s) = \dfrac{3 + 5s}{4 + 2s}$.

SOL:

By long division,

$$4+2s \overline{)\, 3 + 5s \,(} \; \frac{3}{4}$$
$$\underline{3 + \frac{3}{2}s}$$
$$\frac{7}{2}s$$

$$Y(s) = \frac{3}{4} + \frac{\frac{7}{2}s}{4+2s} = Y_1(s) + Y_2(s)$$

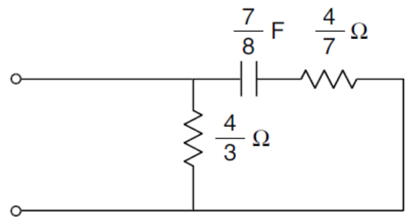
$Y_1(s) = \frac{3}{4}$ represents the admittance of a resistor of value $\frac{4}{3}\ \Omega$.

$$Z_2(s) = \frac{1}{Y_2(s)} = \frac{4+2s}{\frac{7}{2}s} = \frac{8+4s}{7s} = \frac{8}{7s} + \frac{4}{7} = Z_3(s) + Z_4(s)$$

$Z_3(s) = \frac{8}{7s}$ represents the impedance of a capacitor of value $\frac{7}{8}$F.

$Z_4(s) = \frac{4}{7}$ represents the impedance of a resistor of value $\frac{4}{7}\ \Omega$.

The impedances are connected in the series branches, whereas the admittances are connected in the parallel branches. The network is shown in Fig.



SYNTHESIS OF NETWORKS BY FOSTER'S AND CAUER'S METHODS

The Foster's method of network synthesis uses the partial fraction expansion of the driving point immittance function. Foster is of two types: Foster form-I(Series) and Foster form-II(Parallel). When the driving point function is an impedance, it is referred to as Foster form-I. Foster form-II is used for driving point admittance function.

The Cauer's methods (Ladder) employ continued expansion approach to synthesize a given immittance function. In Cauer form-I, the terms of both numerator and denominator are arranged in descending degree of s. In Cauer form-II, the terms in the numerator and denominator polynomials of the driving point immittance function are arranged in ascending order.

For one port networks, they are synthesized into LC, RC and RL networks.

SYNTHESIS OF ONE PORT 'LC' DRIVING POINT IMMITTANCE FUNCTIONS

Properties of LC driving point immittance functions

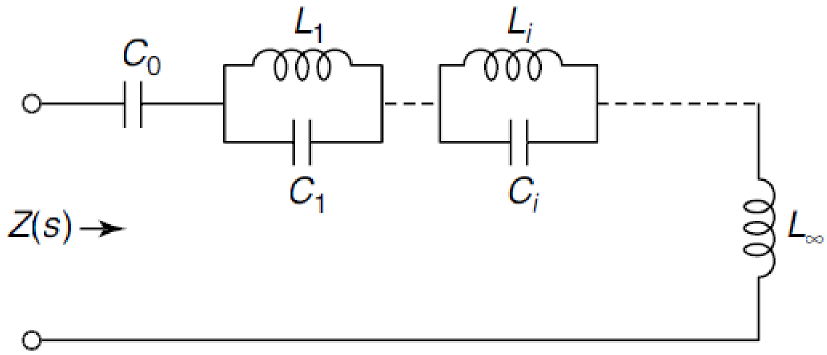
1. It is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the jw-axis.
3. The poles and zeros interlace on the jw-axis.
4. There must be either a zero or a pole at the origin and infinity.
5. The difference between any two successive powers of numerator and denominator polynomials is at most two. There cannot be any missing terms.
6. The highest powers of numerator and denominator polynomials must differ by unity; the lowest powers also differ by unity.

The general form of the partial fraction expansion of LC immittance positive real function is given by

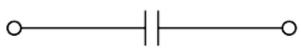
$$\mathrm{F(s)} = \frac{K_0}{s} + \frac{2K_2 s}{s^2+\omega_2{}^2} + \cdots \ldots \ldots \ldots + \frac{2K_j s}{s^2+\omega_j{}^2} + \cdots \ldots . + K_\infty s$$

FOSTER-I FORM (OR) FIRST FOSTER FORM

If the given function F(s) is an impedance function Z(s), then it can be realized in the first Foster form. The network consists of a series capacitor, a number of parallel LC networks and a inductor as shown in the fig.

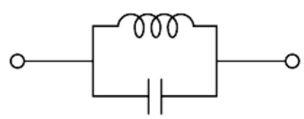


The values of the elements are

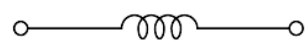


$$C_0 = \frac{1}{K_0}$$

$$L_i = \frac{2K_i}{\omega_i^2}$$



$$C_i = \frac{1}{2K_i}$$



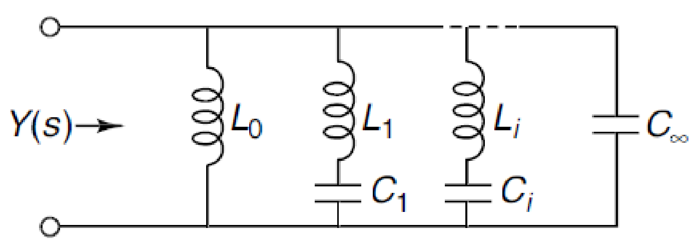
$$L_\infty = K_\infty$$

If Z(s) has no pole at the origin then capacitor C_0 is not present in the network. Similarly, if there is no pole at ∞, inductor L_∞ is not present in the network.

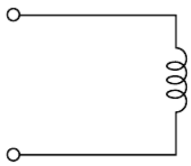
FOSTER-II FORM (OR) SECOND FOSTER FORM

If the given function F(s) is an admittance function Y(s), then it can be realized in the second Foster form as shown in the fig.

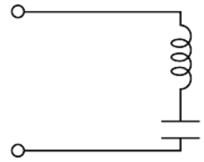
$$F(s) = Y(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2 + \omega_2^2} + \ldots + K_\infty s = Y_1(s) + Y_2(s) + \ldots Y_n(s)$$



The values of the elements are

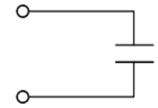


$$L_0 = \frac{1}{K_0}$$



$$L_i = \frac{1}{2K_i}$$

$$C_i = \frac{2K_i}{\omega_i^2}$$



$$C_\infty = K_\infty$$

If Y(s) has no pole at the origin then inductor L_0 is not present. Similarly, if there is no pole at infinity, capacitor C_∞ is not present.

CAUER REALISATION OR LADDER REALISATION

CAUER-I FORM (OR) FIRST CAUER FORM

Since the numerator and denominator polynomials of an *LC* function always differ in degrees by unity, there is always a zero or a pole at $s = \infty$. The Cauer I Form is obtained by successive removal of a pole or a zero at infinity from the function.

Consider an impedance function $Z(s)$ having a pole at infinity.

By removing the pole at infinity, we get

$$Z_2(s) = Z(s) - L_1 s$$

Now, $Z_2(s)$ has a zero at $s = \infty$. If we invert $Z_2(s)$, $Y_2(s)$ will have a pole at $s = \infty$.

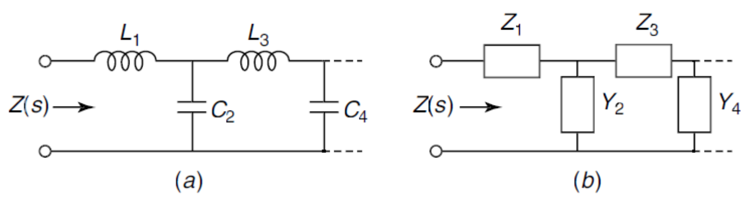
By removing this pole,

$$Y_3(s) = Y_2(s) - C_2 s$$

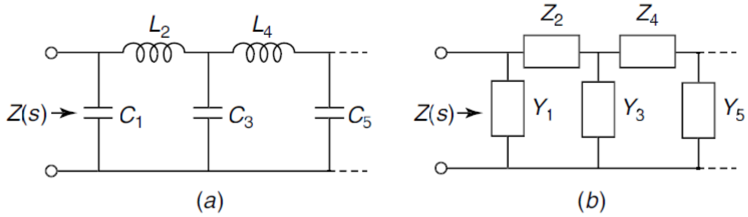
Now $Y_3(s)$ has a zero at $s = \infty$, which we can invert and remove. This process continues until the remainder is zero. Each time we remove a pole, we remove an inductor or a capacitor depending on whether the function is an impedance or an admittance. The impedance $Z(s)$ can be written as a continued fraction expansion.

$$Z(s) = L_1 s + \cfrac{1}{C_2 s + \cfrac{1}{L_3 s + \cfrac{1}{C_4 s + \dots}}}$$

Thus, the final structure is a ladder network whose series arms are inductors and shunt arms are capacitors. The Cauer I network is shown in Fig.



If the impedance function has zero at infinity, i.e., if degree of numerator is less than that of its denominator by unity, the function is first inverted and continued fraction expansion proceeds as usual. In this case, the first element is a capacitor as shown in Fig.

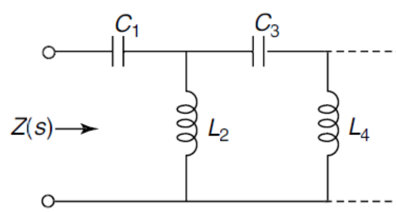


CAUER-II FORM (OR) SECOND CAUER FORM

Since the lowest degrees of numerator and denominator polynomials of *LC* function must differ by unity, there is always a zero or a pole at $s = 0$. The Cauer II form is obtained by successive removal of a pole or a zero at $s = 0$ from the function.

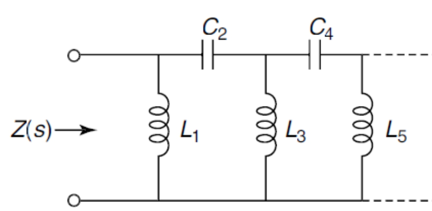
In this method, continued fraction expansion of $Z(s)$ is carried out in terms of poles at the origin by removal of the pole at the origin, inverting the resultant function to create a pole at the origin which is removed and this process is continued until the remainder is zero. To do this, we arrange both numerator and denominator polynomials in ascending order and divide the lowest power of the denominator into the lowest power of the numerator. Then we invert the remainder and divide again. The impedance $Z(s)$ can be written as a continued fraction expansion.

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{L_2 s} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cfrac{1}{L_4 s} + \dots}}}$$



Thus, the final structure is a ladder network whose first element is a series capacitor and second element is a shunt inductor as shown in Fig.

If the impedance function has a zero at the origin then the first element is a shunt inductor and the second element is a series capacitor as shown in Fig.



Thus, the *LC* function $F(s)$ can be realised in four different forms. All these forms have the same number of elements and the number is equal to the number of poles and zeros of $F(s)$ includng any at infinity.

PROBLEMS ON LC FUNCTIONS

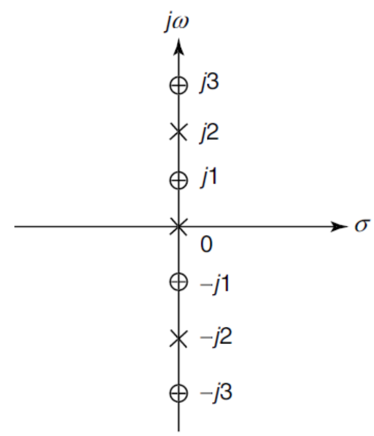
1) Realise the Foster and Cauer forms of the following impedance function

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)}$$

SOL:

The function $Z(s)$ has poles at $s = 0$ and $s = \pm j2$ and zeros at $s = \pm j1$ and $s = \pm j3$ as shown in Fig.

From the pole-zero diagram, it is clear that poles and zeros are simple and lie on the $j\omega$ axis. Poles and zeros are interlaced. Hence, the given function is an LC function.



Foster I Form The Foster I form is obtained by partial-fraction expansion of the impedance function $Z(s)$. But degree of numerator is greater than degree of denominator. Hence, division is first carried out.

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{4s^4+40s^2+36}{s^3+4s}$$

$$s^3+4s\,)\;4s^4+40s^2+36\;(4s$$
$$\underline{4s^4+16s^2}$$
$$24s^2+36$$

$$Z(s) = 4s + \frac{24s^2+36}{s^3+4s} = 4s + \frac{24s^2+36}{s(s^2+4)}$$

By partial-fraction expansion,

$$Z(s) = 4s + \frac{K_0}{s} + \frac{K_1}{s+j2} + \frac{K_1^*}{s-j2} = 4s + \frac{K_0}{s} + \frac{2K_1 s}{s^2+4}$$

where

$$K_0 = sZ(s)\Big|_{s=0} = \frac{4(1)(9)}{4} = 9$$

$$K_1 = \frac{(s^2+4)Z(s)}{2s}\Bigg|_{s^2=-4} = \frac{4(-4+1)(-4+9)}{2(-4)} = \frac{15}{2}$$

$$Z(s) = 4s + \frac{9}{s} + \frac{15s}{s^2+4}$$

The first term represents the impedance of an inductor of 4 H. The second term represents the impedance of a capacitor of $\frac{1}{9}$ F. The third term represents the impedance of a parallel LC network.

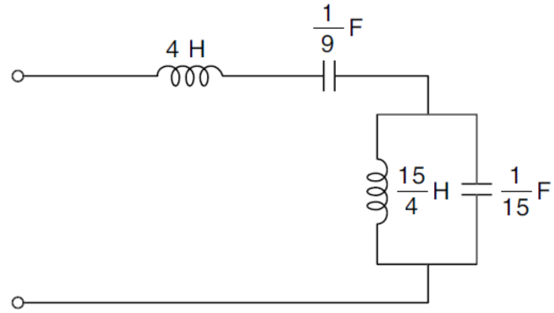
For a parallel LC network,

$$Z_{LC}(s) = \frac{\left(\frac{1}{C}\right)s}{s^2 + \frac{1}{LC}}$$

By direct comparison, $C = \frac{1}{15}$ F

$$L = \frac{15}{4}\text{H}$$

The network is shown in Fig.



Foster II Form The Foster II form is obtained by partial-fraction expansion of the admittance function $Y(s)$.

$$Y(s) = \frac{s(s^2+4)}{4(s^2+1)(s^2+9)}$$

By partial-fraction expansion,

$$Y(s) = \frac{K_1}{s+j1} + \frac{K_1^*}{s-j1} + \frac{K_2}{s+j3} + \frac{K_2^*}{s-j3} = \frac{2K_1 s}{s^2+1} + \frac{2K_2 s}{s^2+9}$$

where

$$K_1 = \frac{(s^2+1)}{2s} Y(s)\bigg|_{s^2=-1} = \frac{(-1+4)}{8(-1+9)} = \frac{3}{64}$$

$$K_2 = \frac{(s^2+9)}{2s} Y(s)\bigg|_{s^2=-9} = \frac{(-9+4)}{8(-9+1)} = \frac{5}{64}$$

$$Y(s) = \frac{\left(\frac{3}{32}\right)s}{s^2+1} + \frac{\left(\frac{5}{32}\right)s}{s^2+9}$$

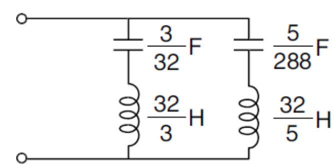
These two terms represent admittance of a series LC network. For a series LC network,

$$Y_{LC}(s) = \frac{\left(\frac{1}{L}\right)s}{s^2 + \frac{1}{LC}}$$

By direct comparison,

$$L_1 = \frac{32}{3}\text{H} \qquad C_1 = \frac{3}{32}\text{F}$$

$$L_2 = \frac{32}{5}\text{H} \qquad C_2 = \frac{5}{288}\text{F}$$



The network is shown in Fig.

Cauer I Form The Cauer I form is obtained from continued fraction expansion about the pole at infinity.

$$Z(s) = \frac{4s^4 + 40s^2 + 36}{s^3 + 4s}$$

Since the degree of the numerator is greater than the degree of the denominator by one, it indicates the presence of a pole at infinity.

By continued fraction expansion,

$$s^3 + 4s \,\big)\, 4s^4 + 40s^2 + 36 \,\big(\, 4s \leftarrow Z$$

$$\underline{4s^4 + 16s^2}$$

$$24s^2 + 36 \,\big)\, s^3 + 4s \,\big(\, \frac{1}{24}s \leftarrow Y$$

$$\underline{s^3 + \frac{3}{2}s}$$

$$\frac{5}{2}s \,\big)\, 24s^2 + 36 \,\big(\, \frac{48}{5}s \leftarrow Z$$

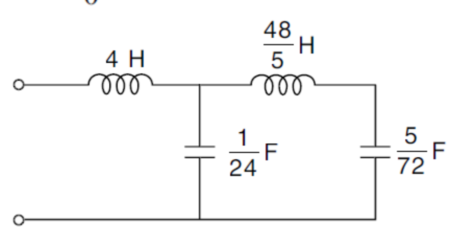
$$\underline{24s^2}$$

$$36 \,\big)\, \frac{5}{2}s \,\big(\, \frac{5}{72}s \leftarrow Y$$

$$\underline{\frac{5}{2}s}$$

$$0$$

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches in a Cauer or ladder realisation. The network is shown in Fig.



Cauer II Form The Cauer II form is obtained from partial-fraction expansion about pole at origin.

$$Z(s) = \frac{4(s^2+1)(s^2+9)}{s(s^2+4)} = \frac{4s^4 + 40s^2 + 36}{s^3 + 4s}$$

The function $Z(s)$ has a pole at origin. Arranging the numerator and denominator polynomials in ascending order of s,

$$Z(s) = \frac{36 + 40s^2 + 4s^4}{4s + s^3}$$

By continued fraction expansion,

$$4s+s^3\,\big)\,36+40s^2+4s^4\,\Big(\frac{9}{s}\leftarrow Z$$

$$\underline{36+\;\;9s^2}$$

$$31s^2+4s^4\,\big)\,4s+s^3\,\Big(\frac{4}{31s}\leftarrow Y$$

$$\underline{4s+\frac{16}{31}s^3}$$

$$\frac{15}{31}s^3\,\big)\,31s^2+4s^4\,\Big(\frac{961}{15s}\leftarrow Z$$

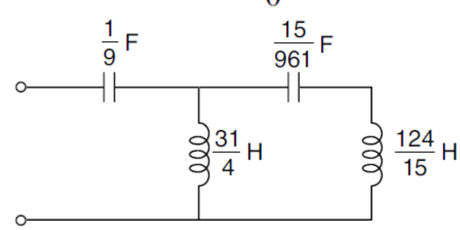
$$\underline{31s^2}$$

$$4s^4\,\big)\,\frac{15}{31}s^3\,\Big(\frac{15}{124s}\leftarrow Y$$

$$\underline{\frac{15}{31}s^3}$$

$$0$$

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches in a Cauer or ladder realisation. The network is shown in Fig.



2) Realise Foster forms of the LC impedance function

$$Z(s)=\frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

SOL:

Foster I Form The Foster I form is obtained by partial-fraction expansion of the impedance function $Z(s)$. Since the degree of the numerator is greater than the degree of the denominator, division is first carried out.

$$Z(s)=\frac{(s^2+1)(s^2+3)}{s(s^2+2)}=\frac{s^4+4s^2+3}{s^3+2s}$$

$$s^3+2s\,\big)\,s^4+4s^2+3\,\big(\,s$$

$$\underline{s^4+2s^2}$$

$$2s^2+3$$

$$Z(s)=s+\frac{2s^2+3}{s^3+2s}=s+\frac{2s^2+3}{s(s^2+2)}$$

By partial-fraction expansion,

$$Z(s) = s + \frac{K_0}{s} + \frac{K_1}{s+j2} + \frac{K_1^*}{s-j2} = s + \frac{K_0}{s} + \frac{2K_1 s}{s^2+2}$$

where
$$K_0 = sZ(s)|_{s=0} = \frac{(1)(3)}{2} = \frac{3}{2}$$
$$K_1 = \frac{(s^2+2)}{2s} Z(s)\bigg|_{s^2=-2} = \frac{(-2+1)(-2+3)}{2(-2)} = \frac{1}{4}$$
$$Z(s) = s + \frac{\left(\frac{3}{2}\right)}{s} + \frac{\left(\frac{1}{2}\right)s}{s^2+2}$$

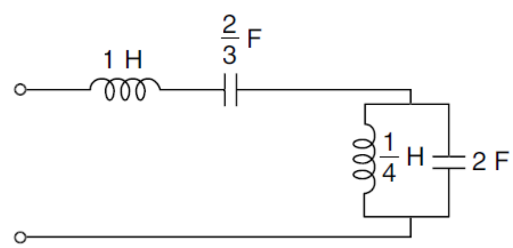
The first term represents the impedance of an inductor of 1 H. The second term represents the impedance of a capacitor of $\frac{2}{3}$ F. The third term represents the impedance of a parallel LC network.

For a parallel LC network,

$$Z_{LC}(s) = \frac{\left(\frac{1}{C}\right)s}{s^2 + \frac{1}{LC}}$$

By direct comparison,

$$C = 2\text{F}$$
$$L = \frac{1}{4}\text{H}$$



The network is shown in Fig.

Foster II Form The Foster II form is obtained by partial-fraction expansion of the admittance function $Y(s)$.

$$Y(s) = \frac{s(s^2+2)}{(s^2+1)(s^2+3)}$$

By partial-fraction expansion,

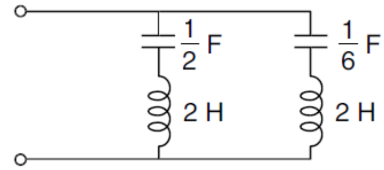
$$Y(s) = \frac{K_1}{s+j1} + \frac{K_1^*}{s-j1} + \frac{K_2}{s+j\sqrt{3}} + \frac{K_2^*}{s-j\sqrt{3}} = \frac{2K_1 s}{s^2+1} + \frac{2K_2 s}{s^2+3}$$

where
$$K_1 = \frac{(s^2+1)}{2s} Y(s)\bigg|_{s^2=-1} = \frac{(-1+2)}{2(-1+3)} = \frac{1}{4}$$
$$K_2 = \frac{(s^2+3)}{2s} Y(s)\bigg|_{s^2=-3} = \frac{-3+2}{2(-3+1)} = \frac{1}{4}$$
$$Y(s) = \frac{\left(\frac{1}{2}\right)s}{s^2+1} + \frac{\left(\frac{1}{2}\right)s}{s^2+3}$$

These two terms represent admittance of a series LC network. For a series LC network,

$$Y_{LC}(s) = \frac{\left(\frac{1}{L}\right)s}{s^2 + \frac{1}{LC}}$$

By direct comparison,



$$L_1 = 2\text{H}, \quad C_1 = \frac{1}{2}\text{F}$$

$$L_2 = 2\text{H}, \quad C_2 = \frac{1}{6}\text{F}$$

The network is shown in Fig.

3) Realise Cauer forms of the following LC impedance function

$$Z(s) = \frac{10s^4 + 12s^2 + 1}{2s^3 + 2s}$$

SOL:

Cauer I Form The Cauer I form is obtained from continued fraction expansion about the pole at infinity.

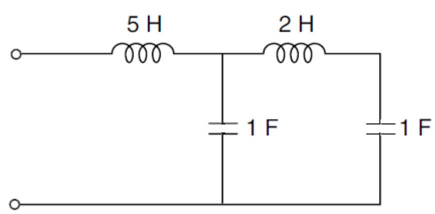
$$Z(s) = \frac{10s^4 + 12s^2 + 1}{2s^3 + 2s}$$

Since the degree of the numerator is greater than the degree of the denominator by one, it indicates the presence of a pole at infinity.

By continued fraction expansion,

$$2s^3 + 2s)\,10s^4 + 12s^2 + 1\,(5s \leftarrow Z$$
$$\underline{10s^4 + 10s^2}$$
$$2s^2 + 1)\,2s^3 + 2s\,(s \leftarrow Y$$
$$\underline{2s^3 + s}$$
$$s)\,2s^2 + 1\,(2s \leftarrow Z$$
$$\underline{2s^2}$$
$$1)\,s\,(s \leftarrow Y$$
$$\underline{s}$$
$$0$$

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches in a Cauer or ladder realisation. The network is shown in Fig.



Cauer II Form The Cauer II form is obtained from continued fraction expansion about the pole at the origin.

$$Z(s) = \frac{10s^4 + 12s^2 + 1}{2s^3 + 2s}$$

The function $Z(s)$ has a pole at the origin. Arranging the numerator and denominator polynomials in ascending order of s,

$$Z(s) = \frac{1 + 12s^2 + 10s^4}{2s + 2s^3}$$

By continued fraction expansion of $Z(s)$,

$$2s + 2s^3 \overline{\big)\, 1 + 12s^2 + 10s^4 \,}\left(\frac{1}{2s} \leftarrow Z\right.$$

$$\underline{1 + \quad s^2}$$

$$11s^2 + 10s^4 \overline{\big)\, 2s + 2s^3 \,}\left(\frac{2}{11s} \leftarrow Y\right.$$

$$\underline{2s + \frac{20}{11}s^3}$$

$$\frac{2}{11}s^3 \overline{\big)\, 11s^2 + 10s^4 \,}\left(\frac{121}{2s} \leftarrow Z\right.$$

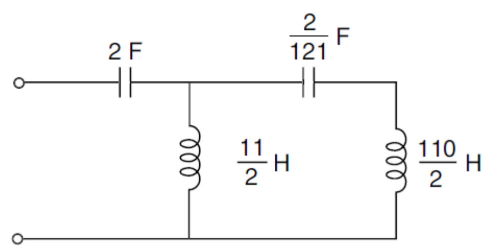
$$\underline{11s^2}$$

$$10s^4 \overline{\big)\, \frac{2}{11}s^3 \,}\left(\frac{2}{110s} \leftarrow Y\right.$$

$$\underline{\frac{2}{11}s^3}$$

$$0$$

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches in Cauer or ladder realisation. The network is shown in Fig.



4) Realise the following network function in Cauer I form

$$Z(s) = \frac{6s^4 + 42s^2 + 48}{s^5 + 18s^3 + 48s}$$

SOL:

The Cauer I form is obtained by continued fraction expansion of $Z(s)$ about the pole at infinity. In the above function, the degree of the numerator is less than the degree of the denominator which indicates the presence of a zero at infinity. The admittance function $Y(s)$ has a pole at infinity. Hence, the continued fraction expansion of $Y(s)$ is carried out.

$$Y(s) = \frac{s^5 + 18s^3 + 48s}{6s^4 + 42s^2 + 48}$$

By continued fraction expansion

$$6s^4 + 42s^2 + s^2 \,\big)\, s^5 + 18s^3 + 48s \,\big(\, \frac{1}{6}s \leftarrow Y$$

$$\underline{s^5 + 7s^3 + 8s}$$

$$11s^3 + 40s \,\big)\, 6s^4 + 42s^2 + 48 \,\big(\, \frac{6}{11}s \leftarrow Z$$

$$\underline{6s^4 + \frac{240}{11}s^2}$$

$$\frac{222}{11}s^2 + 30 \,\big)\, 11s^3 + 40s \,\big(\, \frac{121}{222}s \leftarrow Y$$

$$\underline{11s^3 + \frac{5808}{222}s}$$

$$\frac{3072}{222}s \,\big)\, \frac{222}{11}s^2 + 48 \,\big(\, \frac{49284}{33792}s \leftarrow Z$$

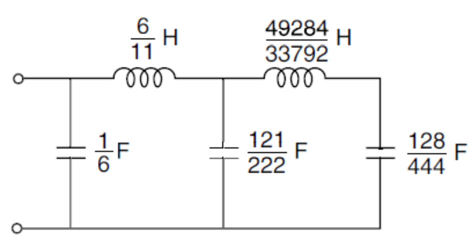
$$\underline{\frac{222}{11}s^2}$$

$$48 \,\big)\, \frac{3072}{222}s \,\big(\, \frac{128}{444}s \leftarrow Y$$

$$\underline{\frac{3072}{222}s}$$

$$0$$

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches in a Cauer or ladder realisation. The network is shown in Fig.



5) Realise Cauer II form of the function

$$Z(s) = \frac{s(s^4 + 3s^2 + 1)}{3s^4 + 4s^2 + 1}$$

SOL:

The Cauer II form is obtained by continued fraction expansion about the pole at the origin. The given function has a zero at the origin. The admittance function $Y(s)$ has a pole at origin. Hence, the continued fraction expansion of $Y(s)$ is carried out. Arranging the polynomials in ascending order of s,

$$Y_{LC}(s) = \frac{3s^4 + 4s^2 + 1}{s^5 + 3s^3 + s} = \frac{1 + 4s^2 + 3s^4}{s + 3s^3 + s^5}$$

By continued fraction expansion of $Y(s)$, we have

$$s + 3s^3 + s^5 \Big) 1 + 4s^2 + 3s^4 \Big(\frac{1}{s} \leftarrow Y$$

$$\underline{1 + 3s^2 + \; s^4}$$

$$s^2 + 2s^4 \Big) s + 3s^3 + s^5 \Big(\frac{1}{s} \leftarrow Z$$

$$\underline{s + 2s^3}$$

$$s^3 + s^5 \Big) s^2 + 2s^4 \Big(\frac{1}{s} \leftarrow Y$$

$$\underline{s^2 + \; s^4}$$

$$s^4 \Big) s^3 + s^5 \Big(\frac{1}{s} \leftarrow Z$$

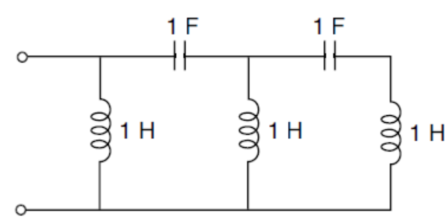
$$\underline{s^3}$$

$$s^5 \Big) s^4 \Big(\frac{1}{s} \leftarrow Y$$

$$\underline{s^4}$$

$$0$$

The impedances are connected in the series branches whereas the admittances are connected in the parallel branches in a Cauer or ladder realisation. The network is shown in Fig.



SYNTHESIS OF ONE PORT 'RC' IMPEDANCE / 'RL' ADMITTANCE FUNCTIONS

RC driving point impedance/ RL admittance functions have following properties:

1. The poles and zeros are simple and are located on the negative real axis of the s plane.
2. The poles and zeros are interlaced.
3. The lowest critical frequency nearest to the origin is a pole.
4. The highest critical frequency farthest to the origin is a zero.
5. Residues evaluated at the poles of Z_{RC} (s) are real and positive.
6. The slope $\frac{d}{d\sigma} Z_{RC}$ is negative.
7. $Z_{RC}(\infty) < Z_{RC}(0)$.

RC functions can also be realised in four different ways. The impedance function of RC networks is given by,

$$Z(s) = \frac{H(s+\sigma_1)(s+\sigma_3)\ldots}{s(s+\sigma_2)\ldots}$$

FOSTER-I FORM (OR) FIRST FOSTER FORM

The Foster I form is obtained by partial-fraction expansion of $Z(s)$.

$$Z(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

where $K_0, K_1, K_2, \ldots K_\infty$ are residues of $Z(s)$.

$$K_o = sZ(s)\Big|_{s=0}$$

$$K_i = (s+\sigma_i)Z(s)\Big| s = -\sigma_i$$

$$K_\infty = \frac{Z(s)}{s}\Bigg|_{s\to\infty}$$

The first term $\frac{K_0}{s}$ represents the impedance of a capacitor of $\frac{1}{K_o}$ farads.

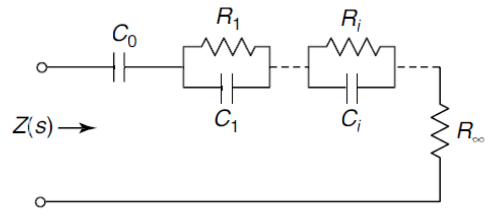
The last term K_∞ represents the impedance of a resistor of K_∞ ohms.

The remaining terms, i.e., $\frac{K_i}{s+\sigma_i}$ represent the impedance of the parallel combination of resistor R_i and capacitor C_i. For parallel combinaton of R_i and C_i,

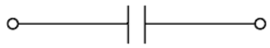
$$Z(s) = \frac{R_i\left(\frac{1}{C_i s}\right)}{R_i + \frac{1}{C_i s}} = \frac{K_i}{s+\sigma_i}$$

$$R_i = \frac{K_i}{\sigma_i} \text{ and } C_i = \frac{1}{K_i}$$

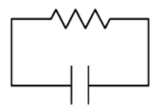
The network corresponding to the Foster-I form is shown in Fig.



The values of the elements are



$$C_0 = \frac{1}{K_0}$$



$$Ri = \frac{K_i}{\sigma_i}$$

$$C_i = \frac{1}{K_i}$$

$$R_\infty = K_\infty$$

FOSTER-II FORM (OR) SECOND FOSTER FORM

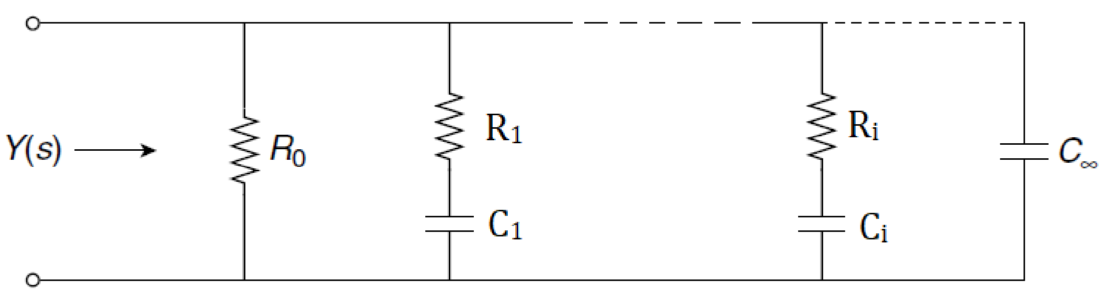
The second Foster form is obtained by expanding the specified RC admittance function in partial fraction.

The Foster-II form is obtained by partial fraction expansion of $Y(s)$. Since $Y(s) = \frac{1}{Z(s)}$ has negative residue at its pole, Foster II form is obtained by expanding $\frac{Y(s)}{s}$.

$$\frac{Y(s)}{s} = \frac{K_o}{s} + \sum_{i=1}^{n} \frac{K_i}{(s+\sigma_i)} + K_\infty$$

$$Y(s) = K_0 + \frac{K_1 s}{s+\sigma_1} + \cdots + \frac{K_i s}{s+\sigma_i} + \cdots + K_\infty s$$

Then the above equation can be realized as RC admittance network as shown in the fig.



Where

$$R_0 = \frac{1}{K_0} \qquad R_1 = \frac{1}{K_1} \qquad C_1 = \frac{K_1}{\sigma_1}$$

$$R_i = \frac{1}{K_i} \qquad C_i = \frac{K_1}{\sigma_i} \qquad C_\infty = K_\infty$$

CAUER-I FORM (OR) FIRST CAUER FORM

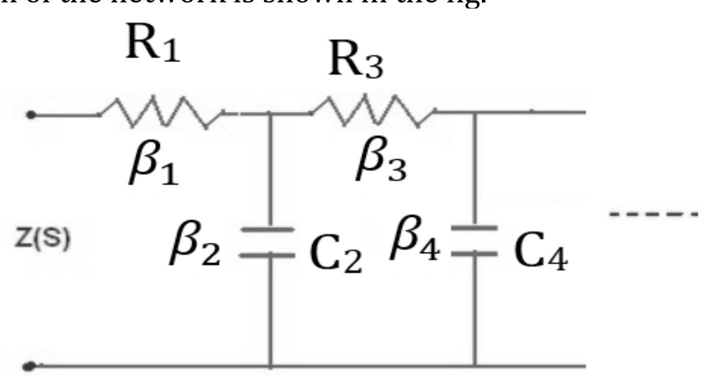
The first Cauer from is obtained by expanding the RC impedance function in continued fraction about infinity. The impedance function in partial fraction is given by

$$Z(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

The continued fraction expansion is given by

$$Z(s) = \beta_1 + \cfrac{1}{\beta_2 s + \cfrac{1}{\beta_3 + \cfrac{1}{\beta_4 s + \cdots \ldots\ldots}}}$$

The first Cauer form of the network is shown in the fig.



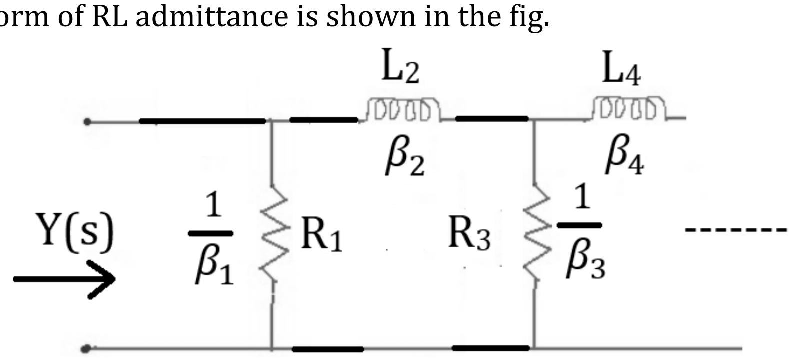
If Z(s) represents an admittance function, then the first Cauer form represents RL admittance realization. The admittance function in partial fraction is given by

$$Y(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

The continued fraction expansion is given by

$$Y(s) = \beta_1 + \cfrac{1}{\beta_2 s + \cfrac{1}{\beta_3 + \cfrac{1}{\beta_4 s + \cdots \ldots\ldots}}}$$

The first Cauer form of RL admittance is shown in the fig.



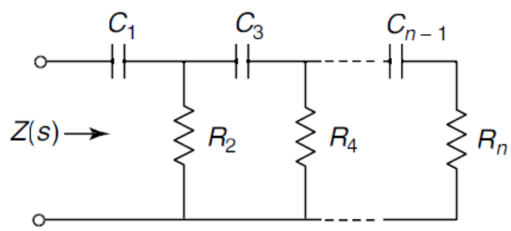
CAUER-II FORM (OR) SECOND CAUER FORM

The Cauer II form is obtained by removal of the pole from the impedance function at the origin. This is the same as a continued fraction expansion of an impedance function about the origin. If the given impedance function has a pole at the origin, it is removed as a capacitor C_1. The reciprocal of the remainder function has a minimum value at s = 0 which is removed as a constant of resistor R_2. If the original impedance has no pole at the origin, then the first capacitor is absent and the process is repeated with the removal of the constant corresponding to the resistor R_2.

The impedance $Z(s)$ can be written as a continued fraction expansion.

$$Z(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cfrac{1}{R_4} + \dots}}}$$

The network is shown in Fig.

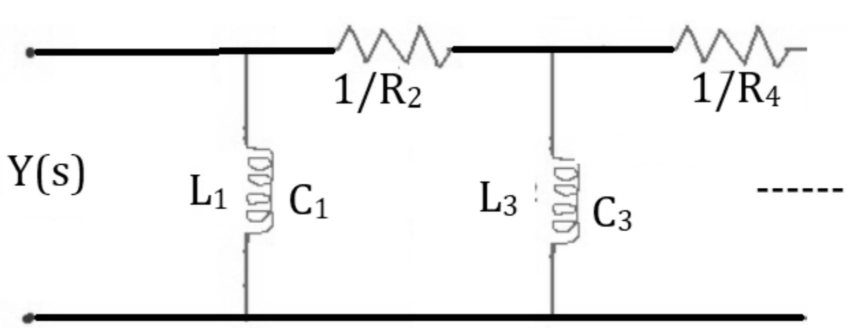


If Z(s) represents an admittance function, then the second Cauer form represents RL admittance realization.

The continued fraction expansion is given by

$$Y(s) = \frac{1}{C_1 s} + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{\cfrac{1}{C_3 s} + \cfrac{1}{\cfrac{1}{R_4} + \dots}}}$$

The second Cauer form of RL admittance is shown in the fig.



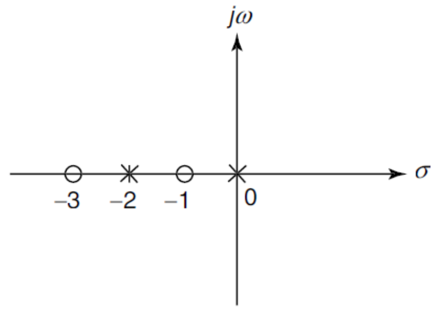
PROBLEMS ON RC NETWORKS

1) Realise the Foster and Cauer forms of the impedance function

$$Z(s)=\frac{(s+1)(s+3)}{s(s+2)}$$

SOL:

The function $Z(s)$ has poles at $s=0$ and $s=-2$ and zeros at $s=-1$ and $s=-3$ as shown in Fig.



From the pole-zero diagram, it is clear that poles and zeros are simple and lie on the negative real axis. The poles and zeros are interlaced and the lowest critical frequency nearest to the origin is a pole. Hence, the function $Z(s)$ is an RC function.

Foster I Form The Foster I form is obtained by partial fraction expansion of impedance function $Z(s)$. Since the degree of the numerator is greater than the degree of the denominator, division is first carried out.

$$Z(s)=\frac{s^2+4s+3}{s^2+2s}$$

$$s^2+2s\,\big)\,s^2+4s+3\,\big(\,1$$
$$\underline{s^2+2s}$$
$$2s+3$$

$$Z(s)=1+\frac{2s+3}{s^2+2s}=1+\frac{2s+3}{s(s+2)}$$

By partial-fraction expansion,

$$Z(s)=1+\frac{K_1}{s}+\frac{K_2}{s+2}$$

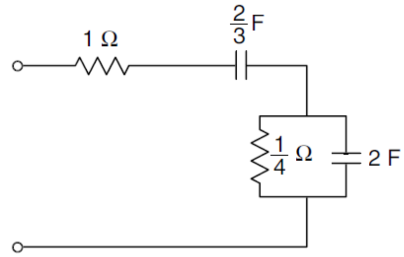
where $$K_1=sZ(s)\big|_{s=0}=\frac{(1)(3)}{2}=\frac{3}{2}$$

$$K_2=(s+2)Z(s)\big|_{s=-2}=\frac{(-2+1)(-2+3)}{-2}=\frac{1}{2}$$

$$Z(s)=1+\frac{\frac{3}{2}}{s}+\frac{\frac{1}{2}}{s+2}$$

The first term represents the impedance of a resistor of 1 Ω. The second term represents the impedance of a capacitor of $\frac{2}{3}$ F. The third term represents the impedance of parallel RC circuit for which

$$Z_{RC}(s) = \frac{\frac{1}{C_i}}{s + \frac{1}{R_i C_i}}$$



By direct comparison,

$$R = \frac{1}{4}\,\Omega$$
$$C = 2\text{ F}$$

The network is shown in Fig.

Foster II Form The Foster II form is obtained by the partial-fraction expansion of admittance function $\frac{Y(s)}{s}$.

$$Y(s) = \frac{1}{Z(s)} = \frac{s(s+2)}{(s+1)(s+3)}$$

$$\frac{Y(s)}{s} = \frac{s+2}{(s+1)(s+3)}$$

By partial-fraction expansion,

$$\frac{Y(s)}{s} = \frac{K_1}{s+1} + \frac{K_2}{s+3}$$

where

$$K_1 = (s+1)\frac{Y(s)}{s}\bigg|_{s=-1} = \frac{(-1+2)}{(-1+3)} = \frac{1}{2}$$

$$K_2 = (s+3)\frac{Y(s)}{s}\bigg|_{s=-3} = \frac{(-3+2)}{(-3+1)} = \frac{1}{2}$$

$$\frac{Y(s)}{s} = \frac{\frac{1}{2}}{s+1} + \frac{\frac{1}{2}}{s+3}$$

$$Y(s) = \frac{\frac{1}{2}s}{s+1} + \frac{\frac{1}{2}s}{s+3}$$

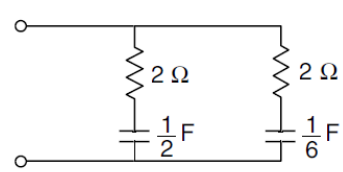
These two terms represent the admittance of a series RC circuit. For a series RC circuit.

$$Y_{RC}(s) = \frac{\left(\frac{1}{R_i}\right)s}{s + \frac{1}{R_i C_i}}$$

By direct comparison,

$$R_1 = 2\,\Omega, \quad C_1 = \frac{1}{2}\text{ F}$$

$$R_2 = 2\,\Omega, \quad C_2 = \frac{1}{6}\text{ F}$$



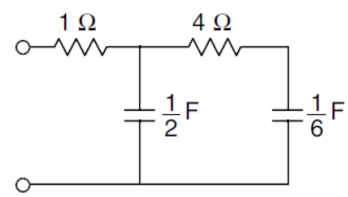
The network is shown in Fig.

Cauer I Form The Cauer I form is obtained by continued fraction expansion about the pole at infinity.

$$Z(s) = \frac{s^2 + 4s + 3}{s^2 + 2s}$$

By continued fraction expansion,

$$s^2 + 2s \Big) s^2 + 4s + 3 \Big(1 \leftarrow Z$$
$$\underline{s^2 + 2s}$$
$$2s + 3 \Big) s^2 + 2s \Big(\frac{1}{2}s \leftarrow Y$$
$$\underline{s^2 + \frac{3}{2}s}$$
$$\frac{1}{2}s \Big) 2s + 3 \Big(4 \leftarrow Z$$
$$\underline{2s}$$
$$3 \Big) \frac{1}{2}s \Big(\frac{1}{6}s \leftarrow Y$$
$$\underline{\frac{1}{2}s}$$
$$0$$



The impedances are connected in the series branches whereas admittances are connected in the parallel branches. The network is shown in Fig.

Cauer II Form The Cauer II form is obtained from continued fraction expansion about the pole at the origin. Arranging the numerator and denominator polynomials of $Z(s)$ in ascending order of s,

$$Z(s) = \frac{3 + 4s + s^2}{2s + s^2}$$

By continued fraction expansion,

$$2s+s^2\,\big)\,3+4s+s^2\,\big(\,\frac{3}{2s} \leftarrow Z$$

$$\underline{3+\frac{3}{2}s}$$

$$\frac{5}{2}s+s^2\,\big)\,2s+s^2\,\big(\,\frac{4}{5} \leftarrow Y$$

$$\underline{2s+\frac{4}{5}s^2}$$

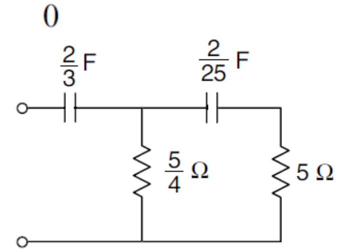
$$\frac{1}{5}s^2\,\big)\,\frac{5}{2}s+s^2\,\big(\,\frac{25}{2s} \leftarrow Z$$

$$\underline{\frac{5}{2}s}$$

$$s^2\,\big)\,\frac{1}{5}s^2\,\big(\,\frac{1}{5} \leftarrow Y$$

$$\underline{\frac{1}{5}s^2}$$

$$0$$



The impedances are connected in the series branches whereas admittances are connected in the parallel branches. The network is shown in Fig.

2) Realise Foster forms of the following RC impedance function

$$Z(s)=\frac{2(s+2)(s+4)}{(s+1)(s+3)}$$

SOL:

Foster I Form The Foster I form is obtained by the partial-fraction expansion of the impedance function $Z(s)$. Since the degree of the numerator is equal to the degree of the denominator, division is carried out first.

$$Z(s)=\frac{2s^2+12s+16}{s^2+4s+3}$$

$$s^2+4s+3\,\big)\,2s^2+12s+16\,\big(\,2$$

$$\underline{2s^2+\;8s+\;6}$$

$$4s+10$$

$$Z(s) = 2 + \frac{4s+10}{s^2+4s+3} = 2 + \frac{4s+10}{(s+1)(s+3)}$$

By partial-fraction expansion,

$$Z(s) = 2 + \frac{K_1}{s+1} + \frac{K_2}{s+3}$$

where

$$K_1 = (s+1)Z(s)\Big|_{s=-1} = \frac{2(-1+2)(-1+4)}{(-1+3)} = 3$$

$$K_2 = (s+3)Z(s)\Big|_{s=-3} = \frac{2(-3+2)(-3+4)}{(-3+1)} = 1$$

$$Z(s) = 2 + \frac{3}{s+1} + \frac{1}{s+3}$$

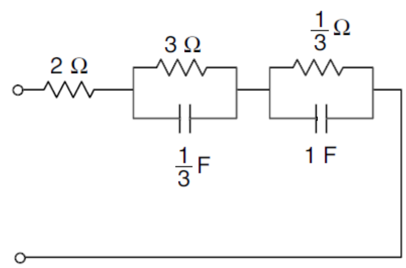
The first term represents the impedance of a resistor of 2 Ω. The remaining terms represent the impedance of a parallel *RC* circuit for which

$$Z_{RC}(s) = \frac{\frac{1}{C_i}}{s + \frac{1}{R_i C_i}}$$

By direct comparison,

$$R_1 = 3\,\Omega, \quad C_1 = \frac{1}{3}\,\text{F}$$

$$R_2 = \frac{1}{3}\,\Omega, \quad C_2 = 1\,\text{F}$$



The network is shown in Fig.

Foster II Form The Foster II form is obtained by partial-fraction expansion of admittance function $\frac{Y(s)}{s}$.

$$Y(s) = \frac{(s+1)(s+3)}{2(s+2)(s+4)}$$

$$\frac{Y(s)}{s} = \frac{(s+1)(s+3)}{2s(s+2)(s+4)}$$

By partial-fraction expansion,

$$\frac{Y(s)}{s} = \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

where

$$K_0 = s\frac{Y(s)}{s}\Big|_{s=0} = \frac{(1)(3)}{(2)(2)(4)} = \frac{3}{16}$$

$$K_1 = (s+2)\frac{Y(s)}{s}\Big|_{s=-2} = \frac{(-2+1)(-2+3)}{2(-2)(-2+4)} = \frac{(-1)(1)}{2(-2)(2)} = \frac{1}{8}$$

$$K_2 = (s+4)\frac{Y(s)}{s}\bigg|_{s=-4} = \frac{(-4+1)(-4+3)}{2(-4)(-4+2)} = \frac{(-3)(-1)}{2(-4)(-2)} = \frac{3}{16}$$

$$\frac{Y(s)}{s} = \frac{\frac{3}{16}}{s} + \frac{\frac{1}{8}}{s+2} + \frac{\frac{3}{16}}{s+4}$$

$$Y(s) = \frac{3}{16} + \frac{\frac{1}{8}s}{s+2} + \frac{\frac{3}{16}s}{s+4}$$

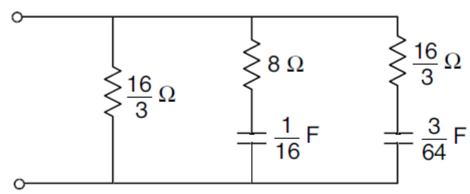
The first term represents the admittance of a resistor of $\frac{16}{3}\,\Omega$. The other two terms represent the admittance of a series RC circuit. For a series RC circuit.

$$Y_{RC}(s) = \frac{\left(\frac{1}{R_i}\right)s}{s + \frac{1}{R_iC_i}}$$

By direct comparison,

$$R_1 = 8\,\Omega, \qquad C_1 = \frac{1}{16}\text{ F}$$

$$R_2 = \frac{16}{3}\,\Omega, \qquad C_2 = \frac{3}{64}\text{ F}$$



The network is shown in Fig.

3) Obtain the Cauer forms of the RC impedance function

$$Z(s) = \frac{(s+2)(s+6)}{2(s+1)(s+3)}$$

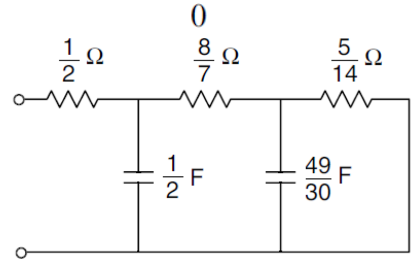
SOL:

Cauer I Form The Cauer I form is obtained by continued fraction expansion about the pole at infinity.

$$Z(s) = \frac{(s+2)(s+6)}{2(s+1)(s+3)} = \frac{s^2+8s+12}{2s^2+8s+6}$$

By continued fraction expansion,

$$
\begin{aligned}
&2s^2+8s+6\,\big)\,s^2+8s+12\,\Big(\tfrac{1}{2} \leftarrow Z\\
&\qquad\qquad \underline{s^2+4s+\ 3}\\
&\qquad\qquad 4s+9\,\big)\,2s^2+8s+6\,\Big(\tfrac{1}{2}s \leftarrow Y\\
&\qquad\qquad\qquad \underline{2s^2+\tfrac{9}{2}s}\\
&\qquad\qquad\qquad \tfrac{7}{2}s+6\,\big)\,4s+9\,\Big(\tfrac{8}{7} \leftarrow Z\\
&\qquad\qquad\qquad\qquad \underline{4s+\tfrac{48}{7}}\\
&\qquad\qquad\qquad\qquad \tfrac{15}{7}\,\big)\,\tfrac{7}{2}s+6\,\Big(\tfrac{49}{30}s \leftarrow Y\\
&\qquad\qquad\qquad\qquad\qquad \underline{\tfrac{7}{2}s}\\
&\qquad\qquad\qquad\qquad\qquad 6\,\big)\,\tfrac{15}{7}\,\Big(\tfrac{5}{14} \leftarrow Z\\
&\qquad\qquad\qquad\qquad\qquad\qquad \underline{\tfrac{15}{7}}\\
&\qquad\qquad\qquad\qquad\qquad\qquad 0
\end{aligned}
$$



The impedances are connected in the series branches whereas the admittances are connected in the parallel branches. The network is shown in Fig.

Cauer II Form The Cauer II form is obtained by continued fraction expansion about the pole at the origin. Arranging the polynomials in ascending order of s,

$$Z(s) = \frac{12+8s+s^2}{6+8s+2s^2}$$

By continued fraction expansion,

$$
\begin{aligned}
&6+8s+2s^2\,\big)\,12+8s+s^2\,\Big(\,2\\
&\qquad\qquad \underline{12+16s+4s^2}\\
&\qquad\qquad\quad -8s-3s^2
\end{aligned}
$$

Since negative term results, continued fraction expansion of $Y(s)$ is carried out.

$$Y(s) = \frac{6+8s+2s^2}{12+8s+s^2}$$

By continued fraction expansion,

$$12+8s+s^2 \,\big)\, 6+8s+2s^2 \,\big(\, \frac{1}{2} \leftarrow Y$$

$$\underline{6+4s+\frac{1}{2}s^2}$$

$$4s+\frac{3}{2}s^2 \,\big)\, 12+8s+s^2 \,\big(\, \frac{3}{s} \leftarrow Z$$

$$\underline{12+\frac{9}{2}s}$$

$$\frac{7}{2}s+s^2 \,\big)\, 4s+\frac{3}{2}+s^2 \,\big(\, \frac{8}{7} \leftarrow Y$$

$$\underline{4s+\frac{8}{7}s^2}$$

$$\frac{5}{14}s^2 \,\big)\, \frac{7}{2}s+s^2 \,\big(\, \frac{49}{5s} \leftarrow Z$$

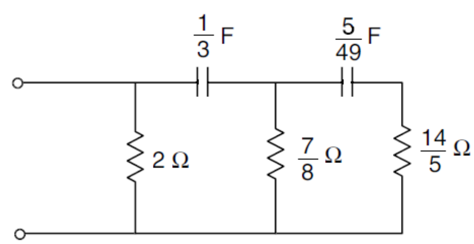
$$\underline{\frac{7}{2}s}$$

$$s^2 \,\big)\, \frac{5}{14}s^2 \,\big(\, \frac{5}{14} \leftarrow Y$$

$$\underline{\frac{5}{14}s}$$

$$0$$

The impedances are connected in the series branches, whereas the admittances are connected in the parallel branches. The network is shown in Fig.



SYNTHESIS OF ONE PORT 'RL' IMPEDANCE / 'RC' ADMITTANCE FUNCTIONS

RL driving point impedance / RC admittance functions have following properties:

1. The poles and zeros are simple and are located on the negative real axis of the s plane.
2. The poles and zeros are interlaced.
3. The lowest critical frequency is a zero which may be at s = 0.
4. The highest critical frequency is a pole which may be at infinity.
5. Residues evaluated at the poles of $Z_{RL}(s)$ are real and negative while that of $\frac{Z_{RL}(s)}{s}$ are real and positive.
6. The slope $\frac{d}{d\sigma}Z_{RL}$ is positive.
7. $Z_{RL}(0) < Z_{RL}(\infty)$.

The admittance of an inductor is similar to the impedance of a capacitor. Hence, properties of an *RL* admittance are identical to those of an *RC* impedance and vice-versa, i.e.,

$$Z_{RC}(s) = Y_{RL}(s)$$
$$Z_{RL}(s) = Y_{RC}(s)$$

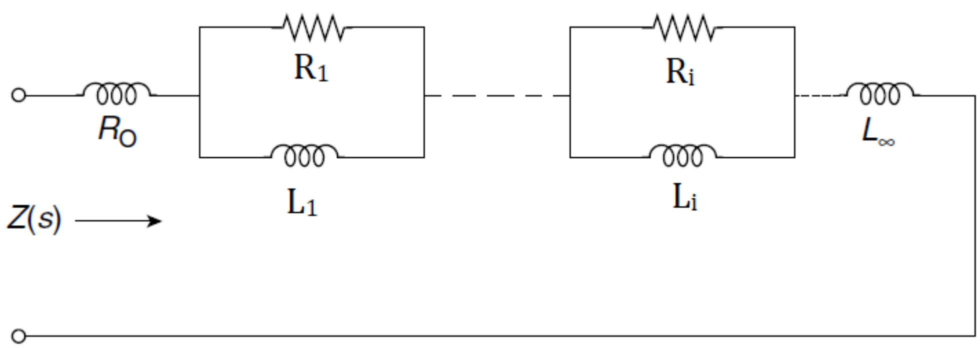
An *RL* admittance can be considered as the dual of an *RC* impedance and vice-versa.

FOSTER-I FORM (OR) FIRST FOSTER FORM

The partial fraction expansion of $Z_{RL}(s)$ is given as follows:

$$Z(s) = K_0 + \frac{K_1 s}{s+\sigma_1} + \cdots + \frac{K_i s}{s+\sigma_i} + \cdots + K_\infty s$$

The general network for RL impedance function in First foster form is shown in the fig.



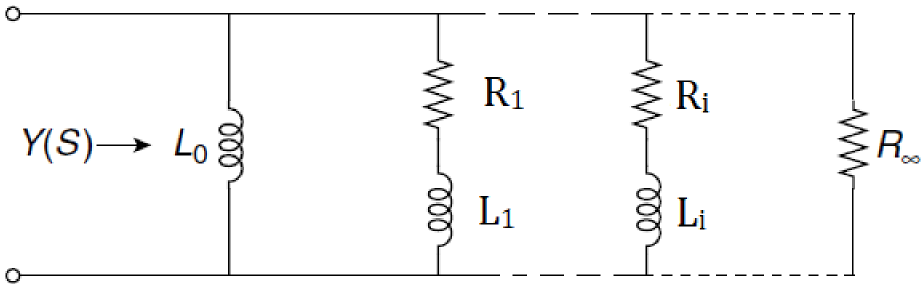
Where

$$R_0 = K_0 \qquad R_1 = K_1 \qquad L_1 = \frac{K_1}{\sigma_1}$$
$$R_i = K_i \qquad L_i = \frac{K_1}{\sigma_i} \qquad \text{-----} \; L_\infty = K_\infty$$

FOSTER-II FORM (OR) SECOND FOSTER FORM

$$Y(s) = \frac{K_0}{s} + \frac{K_1}{s+\sigma_1} + \frac{K_2}{s+\sigma_2} + \ldots + K_\infty$$

Thus, it can be realized as RL admittance and it is shown in the fig.



Where

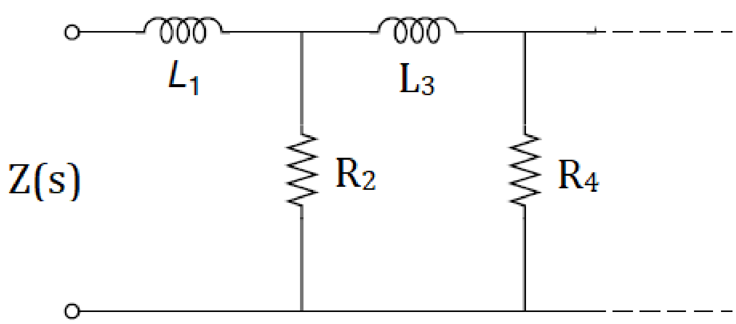
$$L_0 = \frac{1}{K_0},\ R_1 = \frac{\sigma_1}{K_1}, L_1 = \frac{1}{K_1} \ldots \ldots, R_\infty = \frac{1}{K_\infty}$$

CAUER-I FORM (OR) FIRST CAUER FORM

The continued fraction expansion of the impedance function is in the form

$$Z(s) = sL_1 + \cfrac{1}{\cfrac{1}{R_2} + \cfrac{1}{sL_3 + \cfrac{1}{\cfrac{1}{R_4} + \cdots}}}$$

Hence the synthesized network of Z(s) in first Cauer form is shown in the fig.

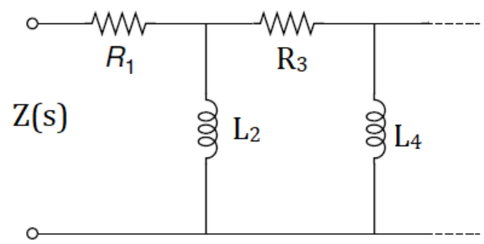


CAUER-II FORM (OR) SECOND CAUER FORM

For second Cauer form, arrange the numerator and denominator polynomials of Z(s) in ascending order, then the continued fraction expansion is in the following form.

$$Z(s) = R_1 + \cfrac{1}{\cfrac{1}{sL_2} + \cfrac{1}{R_3 + \cfrac{1}{\cfrac{1}{sL_4} + \cdots}}}$$

The realization of Z(s) in second Cauer form is shown in the fig.



PROBLEMS ON RL NETWORKS

1) Realise following RL impedance function in Foster-I and Foster-II form.

$$Z(s) = \frac{2(s+1)(s+3)}{(s+2)(s+6)}$$

SOL:

Foster I Form The Foster I form is obtained by partial-fraction expansion of the impedance function $Z(s)$. By partial-fraction expansion,

$$Z(s) = \frac{K_1}{s+2} + \frac{K_2}{s+6}$$

where $$K_1 = (s+2)Z(s)\big|_{s=-2} = \frac{2(-2+1)(-2+3)}{(-2+6)} = -\frac{1}{2}$$

$$K_2 = (s+6)Z(s)\big|_{s=-6} = \frac{2(-6+1)(-6+3)}{(-6+2)} = -\frac{15}{2}$$

Since residues of $Z(s)$ are negative, partial fraction expansion of $\frac{Z(s)}{s}$ is carried out.

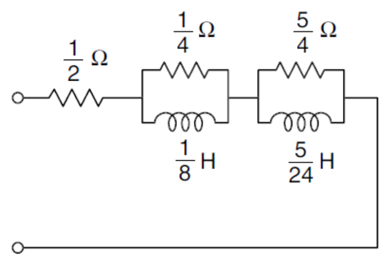
$$\frac{Z(s)}{s} = \frac{2(s+1)(s+3)}{s(s+2)(s+6)}$$

By partial fraction expansion,

$$\frac{Z(s)}{s} = \frac{K_0}{s} + \frac{K_1}{s+2} + \frac{K_2}{s+6}$$

where
$$K_0 = s\frac{Z(s)}{s}\bigg|_{s=0} = \frac{2(1)(3)}{(2)(6)} = \frac{1}{2}$$
$$K_1 = (s+2)\frac{Z(s)}{s}\bigg|_{s=-2} = \frac{2(-2+1)(-2+3)}{(2)(-2+6)} = \frac{1}{4}$$
$$K_2 = (s+6)\frac{Z(s)}{s}\bigg|_{s=-6} = \frac{2(-6+1)(-6+3)}{(-6)(-6+2)} = \frac{5}{4}$$
$$\frac{Z(s)}{s} = \frac{\frac{1}{2}}{s} + \frac{\frac{1}{4}}{s+2} + \frac{\frac{5}{4}}{s+6}$$
$$Z(s) = \frac{1}{2} + \frac{\frac{1}{4}s}{s+2} + \frac{\frac{5}{4}s}{s+6}$$

The first term represents the impedance of the resistor of $\frac{1}{2}\ \Omega$. The other two terms represent the impedance of the parallel RL circuit for which
$$Z_{RL}(s) = \frac{R_i s}{s + \frac{R_i}{L_i}}$$



By direct comparison,
$$R_1 = \frac{1}{4}\ \Omega, \quad L_1 = \frac{1}{8}\ \text{H}$$
$$R_2 = \frac{5}{4}\ \Omega, \quad L_2 = \frac{5}{24}\ \text{H}$$

The network is shown in Fig.

Foster II Form The Foster II form is obtained by partial fraction expansion of $Y(s)$. Since the degree of the numerator is equal to the degree of the denominator, division is first carried out.
$$Y(s) = \frac{(s+2)(s+6)}{2(s+1)(s+3)} = \frac{s^2+8s+12}{2s^2+8s+6}$$
$$2s^2+8s+6\,\big)\, s^2+8s+12\,\big(\,\frac{1}{2}$$
$$\underline{s^2+4s+3}$$
$$4s+9$$
$$Y(s) = \frac{1}{2} + \frac{4s+9}{2s^2+8s+6} = \frac{1}{2} + \frac{4s+9}{2(s+1)(s+3)}$$

By partial-fraction expansion,
$$Y_1(s) = \frac{4s+9}{2(s+1)(s+3)} = \frac{K_0}{s+1} + \frac{K_1}{s+3}$$

where
$$K_0 = (s+1)Y_1(s)\big|_{s=-1} = \frac{(-4+9)}{2(-1+3)} = \frac{5}{4}$$
$$K_1 = (s+3)Y_1(s)\big|_{s=-3} = \frac{(-12+9)}{2(-3+1)} = \frac{3}{4}$$
$$Y(s) = \frac{1}{2} + \frac{\frac{5}{4}}{s+1} + \frac{\frac{3}{4}}{s+3}$$

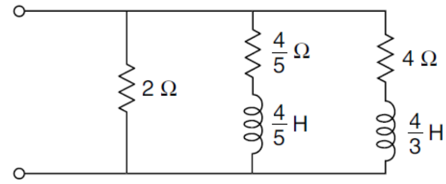
The first term represents the admittance of a resistor of 2 Ω. The other two terms represent the admittance of a series *RL* circuit. For a series *RL* circuit,

$$Y_{RL}(s) = \frac{\frac{1}{L_i}}{s + \frac{R_i}{L_i}}$$

By direct comparison,

$$R_1 = \frac{4}{5}\,\Omega, \quad L_1 = \frac{4}{5}\,\text{H}$$

$$R_2 = 4\,\Omega, \quad L_2 = \frac{4}{3}\,\text{H}$$



The network is shown in Fig.

2) Obtain the Foster I and Cauer I forms of the RL impedance function.

$$Z(s) = \frac{s(s+4)(s+8)}{(s+1)(s+6)}$$

SOL:

Foster I Form The Foster I form is obtained by partial fraction expansion of $\frac{Z(s)}{s}$.

$$\frac{Z(s)}{s} = \frac{(s+4)(s+8)}{(s+1)(s+6)}$$

Since the degree of the numerator is equal to the degree of the denominator, division is first carried out.

$$s^2+7s+6)s^2+12s+32(1$$

$$\underline{s^2+7s+6}$$

$$5s+26$$

$$\frac{Z(s)}{s} = 1 + \frac{5s+26}{s^2+7s+6} = 1 + \frac{5s+26}{(s+1)(s+6)}$$

By partial-fraction expansion,

$$\frac{Z(s)}{s} = 1 + \frac{K_0}{s+1} + \frac{K_1}{s+6}$$

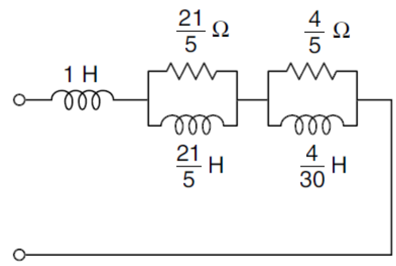
where $$K_0 = \left.\frac{5s+26}{s+6}\right|_{s=-1} = \frac{-5+26}{-1+6} = \frac{21}{5}$$

$$K_1 = \left.\frac{5s+26}{s+1}\right|_{s=-6} = \frac{-30+26}{-6+1} = \frac{4}{5}$$

$$\frac{Z(s)}{s} = 1 + \frac{\frac{21}{5}}{s+1} + \frac{\frac{24}{5}}{s+6}$$

$$Z(s) = s + \frac{\frac{21}{5}s}{s+1} + \frac{\frac{4}{5}s}{s+6}$$

The first term represents the impedance of the inductor of 1 H. The other two terms represent the impedance of a parallel RL circuit for which



$$Z_{RL}(s) = \frac{R_i s}{s + \frac{R_i}{L_i}}$$

By direct comparison,

$$R_1 = \frac{21}{5}\,\Omega, \quad L_1 = \frac{21}{5}\,\text{H}$$

$$R_2 = \frac{4}{5}\,\Omega, \quad L_2 = \frac{4}{30}\,\text{H}$$

The network is shown in Fig.

Cauer I Form The Cauer I form is obtained by continued fraction expansion of $Z(s)$ about the pole at infinity.

$$Z(s) = \frac{s^3 + 12s^2 + 32s}{s^2 + 7s + 6}$$

By continued fraction expansion,

$$s^2 + 7s + 6 \,\big)\, s^3 + 12s^2 + 32s \,\big(\, s \leftarrow Z$$

$$\underline{s^3 + 7s^2 + 6s}$$

$$5s^2 + 26s \,\big)\, s^2 + 7s + 6 \,\big(\, \frac{1}{5} \leftarrow Y$$

$$\underline{s^2 + \frac{26}{5}s}$$

$$\frac{9}{5}s + 6 \,\big)\, 5s^2 + 26s \,\big(\, \frac{25}{9}s \leftarrow Z$$

$$\underline{5s^2 + \frac{50}{3}s}$$

$$\frac{28}{3}s \,\big)\, \frac{9}{5}s + 6 \,\big(\, \frac{27}{140} \leftarrow Y$$

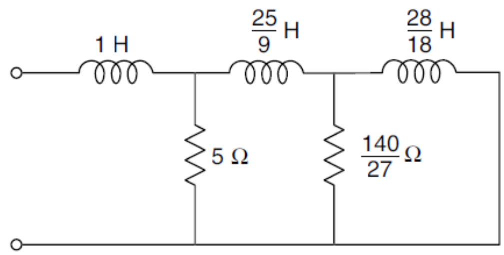
$$\underline{\frac{9}{5}s}$$

$$6 \,\big)\, \frac{28}{3}s \,\big(\, \frac{28}{18}s \leftarrow Z$$

$$\underline{\frac{28}{3}s}$$

$$0$$

The impedances are connected in the series branches, whereas the admittances are connected in the parallel branches. The network is shown in Fig.



3) Find the circuit in second Cauer form of the following function

$$Z(s)=\frac{s^2+4s+3}{s^2+8s+12}$$

SOL:

Given network function is as follows:

$$Z(s)=\frac{s^2+4s+3}{s^2+8s+12}$$

$$Z(0)=\frac{0+0+3}{0+0+12}=\frac{3}{12}=\frac{1}{4}$$

Further, $Z(\infty)$ can be calculated as follows:

$$Z(s)=\frac{s^2\left(1+\frac{4}{s}+\frac{3}{s^2}\right)}{s^2\left(1+\frac{8}{s}+\frac{12}{s^2}\right)}=\frac{1+\frac{4}{s}+\frac{3}{5^2}}{1+\frac{8}{s}+\frac{12}{s^2}}$$

$$Z(\infty)=\frac{1+0+0}{1+0+0}=1$$

$$Z(\infty)>Z(0)$$

Therefore, the given impedance is of R–L type.

Now, Cauer form-II can be obtained as in the following:

$$Z(s)=\frac{3+4s+s^2}{12+8s+s^2}$$

$$12+8s+s^3 \overline{\big)\, 3+4s+s^2} \left(\frac{1}{4} \right.$$

$$3+2s+\frac{s^2}{4}$$

$$2s+\frac{3}{4}s^2 \,\big)\, 12+8s+s^2 \left(\frac{6}{s} \right.$$

$$12+\frac{9}{2}s$$

$$\frac{7s}{2}+s^2 \,\big)\, 2s+\frac{3}{4}s^2 \left(\frac{2}{7s}\times 2s = \frac{4}{7} \right.$$

$$2s+\frac{4s^2}{7}$$

$$\frac{5s^2}{28} \,\big)\, \frac{7s}{2}+s^2 \quad \left(\frac{28}{5s^2}\times\frac{7s}{2} = \frac{98}{5s} \right.$$

$$\frac{7s}{2}$$

$$s^2 \,\big)\, \frac{5}{28}s^2 \left(\frac{5}{28} \right.$$

$$\frac{5}{28}s^2$$

$$\text{x}$$

The Cauer form-II circuit is shown in Figure